# HW 1 Hints

1. Start with the hint. Apply Schwarz inequality to $\int_a^x 1\, u'(s)\, ds$. This should give you an inequality of the form $u(x) \leq (x-a)^{1/2} \left( \int_a^x |u'(s)|^2 ds \right)^{1/2}$.

This holds for all x. The desired inequality follows from this.

2. We will follow a similar argument as in the class notes from 8/30, page III, starting with "multiply by $w$ and integrate". For us, we will multiply our ODE by $u(x)$ and integrate:

$$\int_a^b -(p(x)u'(x))'u(x) + q(x)u'(x)u(x) + r(x)u(x)^2\, dx = \int_a^b f(x)u(x)\, dx.$$

We can bound the term on the right above by $\|f\|_{L^2} \|u\|_{L^2}$ (from Schwarz' inequality). Using the same assumptions / argument in the notes, one can bound the left side below by $\frac{1}{c}\|u\|_{L^2}^2$ for some constant $c > 0$. Thus we have

$$\frac{1}{c}\|u\|_{L^2}^2 \leq \|f\|_{L^2}\|u\|_{L^2},$$

from which the desired inequality follows.

3. (a) Use method of integrating factors.

(b) Start with the hint. If $u$ has a local minimum at $x_0$, then $u'(x_0) = 0$ and $u''(x_0) > 0$. Use the ODE to derive a contradiction.