610 HW 2 Hints

1(a) Use the discretized ODE to show

$$\min(y_{i-1}, y_{i+1}) \le y_i \le \max(y_{i-1}, y_{i+1}).$$

ie that y_i is always between y_{i-1} and y_{i+1}.

1(b) Use the discretized ODE to write

$$0 = \alpha\left(\frac{f_i}{r} - y_i\right) + \beta\left(y_{i-1} - y_i\right) + \gamma\left(y_{i+1} - y_i\right)$$

for some $\alpha, \beta, \gamma > 0$.

1(c) Use the discretized ODE to write

$$y_i = \frac{\alpha}{1+\alpha+\beta} y_{i-1} + \frac{1}{1+\alpha+\beta} y_{i+1} + \frac{\beta}{1+\alpha+\beta}\frac{f_i}{r}$$

for some $\alpha, \beta \ge 0$.

Then try to show

$$\max_i |y_i| \le \frac{\alpha+1}{1+\alpha+\beta} \max_i |y_i| + \frac{\beta}{1+\alpha+\beta}\frac{1}{r}\max_i |f_i|$$

2 Use Taylor's theorem with remainder twice to show

$$u(x+h) = u(x) + hu'(x) + \frac{h^2}{2}u''(x) + \frac{h^3}{6}u'''(x) + \frac{h^4}{24}u^{(4)}(\xi_+)$$

$$u(x-h) = u(x) - hu'(x) + \frac{h^2}{2}u''(x) - \frac{h^3}{6}u'''(x) + \frac{h^4}{24}u^{(4)}(\xi_-)$$

where ξ_+ is some point in $(x, x+h)$ and ξ_- is some point in $(x-h, x)$. Apply this to $x = x_i$. You will need to use the Intermediate Value Theorem at some point to finish the proof.

<u>The Intermediate Value Theorem</u>

If $v : [a,b] \to \mathbb{R}$ is continuous, then for all y between $v(a)$ and $v(b)$, there is some $a \leq \xi \leq b$ st $v(\xi) = y$.

3. Observe that w satisfies the ODE above, so we can apply the inequality to it.

From this, one can show

$$\|u\|_\infty \leq \frac{1}{8}\|f\|_\infty + \left(1 + \frac{r}{8}\right)\max\{|g_0|, |g_1|\}.$$