# MATH 152 Lab 2 Overview

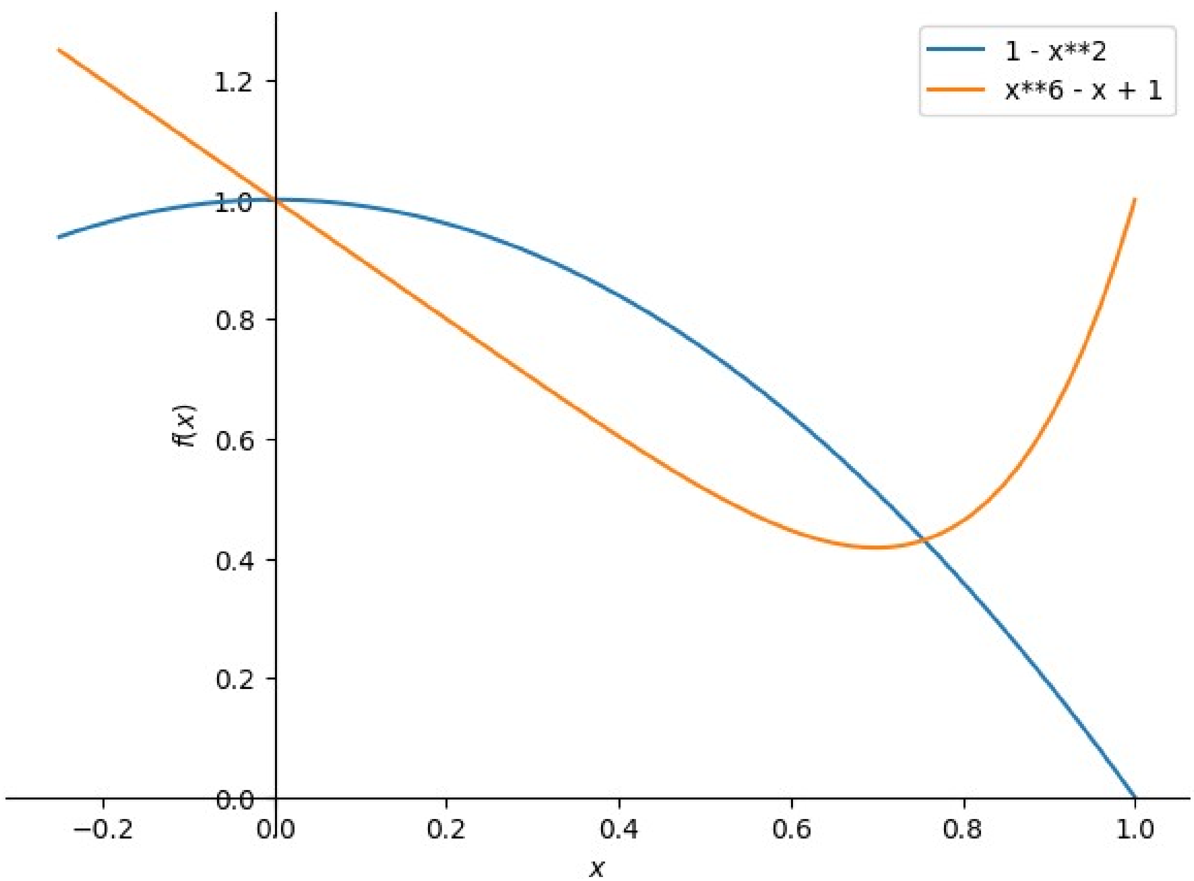
```
import sympy as sp
from sympy.plotting import plot
```

Given the region bounded by the curves $y=1-x^2$ and $y=x^6-x+1$:

1. Find the area of the region.
2. Find the volume of the solid formed by rotating the region about the $x$ axis.
3. Find the value $c$ such that the volume of the region rotated about the line $y=c$ is 5.

First, we visualize the region bounded by the two curves. We do this by using sympy's plotting features.

```
x = sp.symbols('x')
y0 = 1 - x**2
y1 = x**6 - x + 1
plot(y0, y1, (x, -0.25, 1), legend=True)
```



```
<sympy.plotting.backends.matplotlibbackend.matplotlib.MatplotlibBacken
d at 0x79bec24d5750>
```

We see that the two curves intersect at what appears to be $x=0$ and at some point between $x=0.6$ and $x=0.8$. We also see that the curve $y_0=1-x^2$ is larger than the curve $y_1=x^6-x+1$ in between the two intersection points. Call the first intersection point $x_0$ and the second one $x_1$. Then the area of the region bounded by the two curves is computed as

$$A=\int_{x_0}^{x_1} y_0(x)-y_1(x)dx.$$

We will use sympy to solve for $x_0$ and $x_1$ numerically and also compute $A$ numerically.

```
x0 = sp.nsolve(y0 - y1, x, 0)
x1 = sp.nsolve(y0 - y1, x, 0.7)
A = sp.integrate(y0 - y1, (x, x0, x1))

print('The two intersection points are x = {} and x = {}'.format(x0,
x1))
print('The area of the region is {}'.format(A))
```

```
The two intersection points are x = 0 and x = 0.754877666246693
The area of the region is 0.121579206975124
```

Now we find the volume obtained by rotating the region about the $x$ axis. This is computed as

$$V=\pi\int_{x_0}^{x_1} y_1(x)^2-y_2(x)^2dx.$$

Once again, we use sympy to perform this computation.

```
V = sp.pi * sp.integrate(y0**2 - y1**2, (x, x0, x1))
print('The volume of the region rotated about the x axis is
{}'.format(V.evalf()))
```

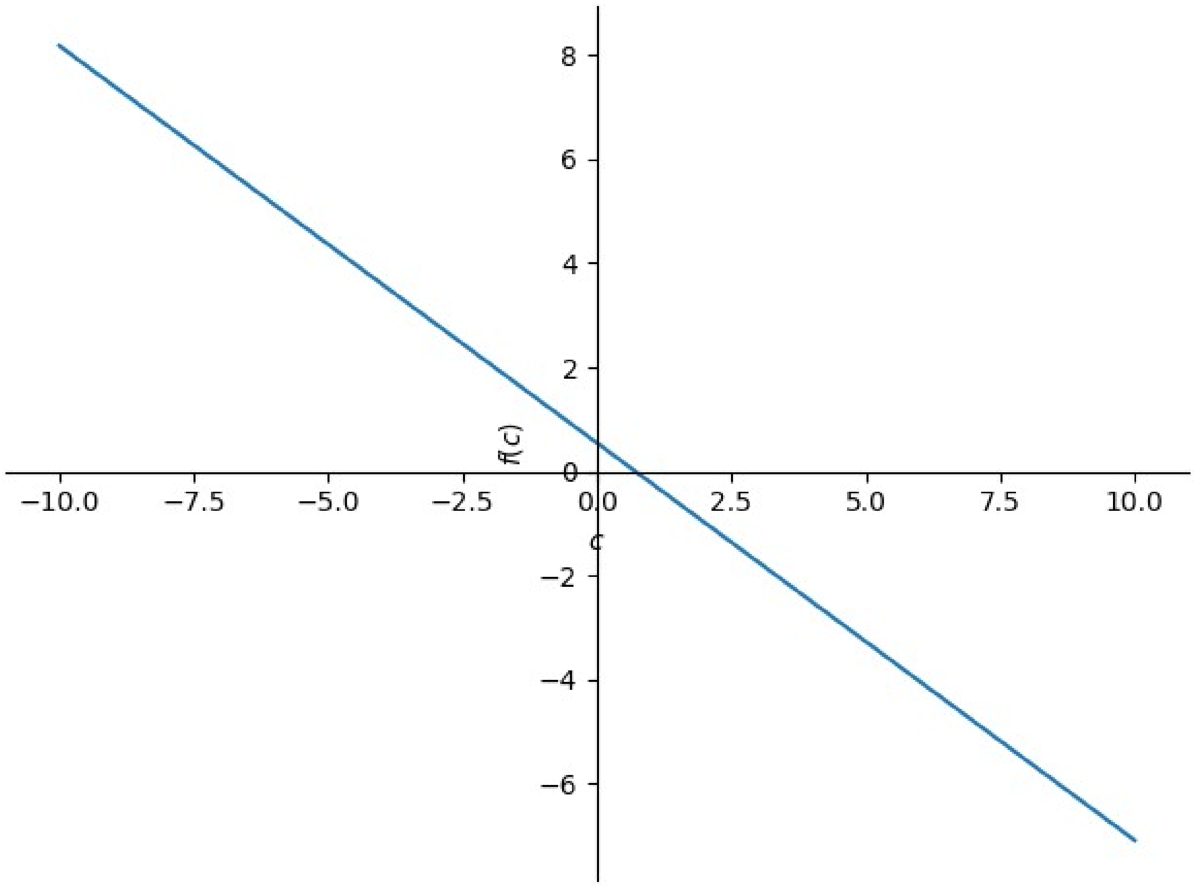
```
The volume of the region rotated about the x axis is 0.544025155285806
```

Now suppose that we want to rotate the region about the line $y=c$ instead of $y=0$. Then the formula for the volume as a function of $c$ is

$$V(c)=\pi\int_{x_0}^{x_1} (y_1(x)-c)^2-(y_2(x)-c)^2dx.$$

Using sympy, we will find the value of $c$ such that $V(c)=5$. Let $c_0$ denote the solution we are looking for. We first visualize $V(c)$ to determine an estimate of $c_0$.

```
c = sp.symbols('c')
V = sp.pi * sp.integrate((y0 - c)**2 - (y1 - c)**2, (x, x0, x1))
plot(V)
```




```
<sympy.plotting.backends.matplotlibbackend.matplotlib.MatplotlibBacken
d at 0x79beb6dc6890>
```

We see that the value $c_0$ such that $V(c_0)=5$ lies somewhere between $c=-7.5$ and $c=-5$. We use sympy to numerically compute $c_0$ with an initial guess of $-6$.

```
c0 = sp.nsolve(V - 5, c, -6)
print('The value of c such that V(c) = 5 is {}'.format(c0))

The value of c such that V(c) = 5 is -5.83315552448461
```

All the answers that we computed were numerical approximations. What if, for some reason, we want exact expressions for the answers? Here is how we obtain them.

```
x0, x1 = sp.solveset(y0 - y1, x, domain=sp.S.Reals)
area = sp.integrate(y0 - y1, (x, x0, x1))
```

```python
volume = sp.pi * sp.integrate((y0**2 - y1)**2, (x, x0, x1))
V = sp.pi * sp.integrate((y0 - c)**2 - (y1 - c)**2, (x, x0, x1))
c0 = sp.solve(V - 5, c)[0]
print('The two intersection points are x = {} and x = {}'.format(x0,
x1))
print('The area of the region is {}'.format(area))
print('The volume of the region rotated about the x axis is
{}'.format(volume))
print('The value of c such that V(c) = 5 is {}'.format(c0))
```

```
The two intersection points are x = 0 and x = -1/3 + 1/(9*(sqrt(69)/18
+ 25/54)**(1/3)) + (sqrt(69)/18 + 25/54)**(1/3)
The area of the region is -(-1/3 + 1/(9*(sqrt(69)/18 + 25/54)**(1/3))
+ (sqrt(69)/18 + 25/54)**(1/3))**3/3 - (-1/3 + 1/(9*(sqrt(69)/18 +
25/54)**(1/3)) + (sqrt(69)/18 + 25/54)**(1/3))**7/7 + (-1/3 +
1/(9*(sqrt(69)/18 + 25/54)**(1/3)) + (sqrt(69)/18 +
25/54)**(1/3))**2/2
The volume of the region rotated about the x axis is pi*(-(-1/3 +
1/(9*(sqrt(69)/18 + 25/54)**(1/3)) + (sqrt(69)/18 + 25/54)**(1/3))**4
- 4*(-1/3 + 1/(9*(sqrt(69)/18 + 25/54)**(1/3)) + (sqrt(69)/18 +
25/54)**(1/3))**7/7 - (-1/3 + 1/(9*(sqrt(69)/18 + 25/54)**(1/3)) +
(sqrt(69)/18 + 25/54)**(1/3))**8/4 - 2*(-1/3 + 1/(9*(sqrt(69)/18 +
25/54)**(1/3)) + (sqrt(69)/18 + 25/54)**(1/3))**11/11 + (-1/3 +
1/(9*(sqrt(69)/18 + 25/54)**(1/3)) + (sqrt(69)/18 +
25/54)**(1/3))**13/13 + 5*(-1/3 + 1/(9*(sqrt(69)/18 + 25/54)**(1/3)) +
(sqrt(69)/18 + 25/54)**(1/3))**9/9 + (-1/3 + 1/(9*(sqrt(69)/18 +
25/54)**(1/3)) + (sqrt(69)/18 + 25/54)**(1/3))**6/3 + (-1/3 +
1/(9*(sqrt(69)/18 + 25/54)**(1/3)) + (sqrt(69)/18 +
25/54)**(1/3))**3/3 + 4*(-1/3 + 1/(9*(sqrt(69)/18 + 25/54)**(1/3)) +
(sqrt(69)/18 + 25/54)**(1/3))**5/5)
The value of c such that V(c) = 5 is (-
33257126344713561273*sqrt(69)*pi*(3*sqrt(69) + 25)**(1/3)/26 -
690636098267697592479*pi*(3*sqrt(69) + 25)**(1/3)/65 -
65864706715978562253*pi*(6*sqrt(69) + 50)**(2/3)/260 -
30496841947104120*sqrt(69)*pi*(6*sqrt(69) + 50)**(2/3) +
1556406374313717433761*2**(1/3)*pi/52 +
936846545599429468251*2**(1/3)*sqrt(69)*pi/260 +
9208780566148800315*(6*sqrt(69) + 50)**(2/3) +
1108606904335476000*sqrt(69)*(6*sqrt(69) + 50)**(2/3))/(pi*(-
10291977232794784961*(3*sqrt(69) + 25)**(1/3) -
1239008459109314423*sqrt(69)*(3*sqrt(69) + 25)**(1/3) +
29234224019520001*2**(2/3)*(3*sqrt(69) + 25)**(2/3) +
3519386997890400*2**(2/3)*sqrt(69)*(3*sqrt(69) + 25)**(2/3) +
21928728770487489968*2**(1/3) +
2639908720121587246*2**(1/3)*sqrt(69)))
```

Since the exact solutions are hard to read, we prefer to solve things numerically.