MATH 152 – PYTHON LAB 9

Directions: Use Python to solve each problem, unless the question states otherwise. (Template link)

1. Given the following power series,

$$f(x) = \sum_{n=0}^{\infty} \frac{(2n)!x^{2n+1}}{2^{2n}(n!)^2(2n+1)}$$

   (a) What is the radius of convergence of the series?

   (Double-check your code when entering this into Python. I recommend entering the numerator and denominator separately, then combining them afterward.)

   (b) It can be shown that this series converges to $\arcsin(x)$ on the inverval $[-1, 1]$. To see this, graph the 1st, 5th, and 10th partial sums and the arcsine function on the same plot with domain $x \in [-1, 1]$.

2. Recall that the **Taylor series** of a function f (centered at $x = a$) is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,$$

   where $f^{(n)}(a)$ is the nth derivative of f evaluated at $x = a$. In SymPy, the nth derivative of a function f can be computed using the command **sp.diff(f,x,n)**, then by substituting $x = a$ we get $f^{(n)}(a)$.

   For each of the following functions, use a **for** loop to compute the 10th degree **Taylor polynomial** (in other words, the partial sum of the Taylor series containing only the terms up to $n = 10$.)

   (a) $f(x) = \cos(x)$, centered at $x = 0$

   (b) $f(x) = \ln(x)$, centered at $x = e$

   (c) $f(x) = \csc(x)$, centered at $x = \pi/2$

   (d) $f(x) = \frac{1}{x}$, centered at $x = 1$

(*Problem 3 is on the back!*)

3. Power series aren't the only way to write a function as an infinite sum of simpler terms! Depending on your choice of engineering major, you'll come across **Fourier series**, which are infinite sums of sines and cosines that represent **periodic** functions (functions that repeat their values at regular intervals). This is especially useful in acoustics.

   Here is one common example, the **sawtooth wave**

$$f(x) = 2\left(x - \left\lfloor x + \frac{1}{2} \right\rfloor\right)$$

   (for any x that isn't $\frac{1}{2}+$ an integer), where $\lfloor \cdot \rfloor$ is the floor function (round down the inside). It can be represented by the Fourier series:

$$\frac{-2}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^k}{k} \sin\left(2\pi k x\right).$$

   (a) Compute the first 5 partial sums of this series. (Set **real** = **True** on your variables.)

   (b) Plot the sawtooth wave along with its first 5 partial sums on the same graph, with x-domain $[0, 3]$. Use **sp.floor** for the floor part of the wave function.