Recitation notes

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1. Section 5.5 Problems

   (a) Problem 27: Compute the indefinite integral

   $$\int (x^2+1)(x^3+3x)^4\,dx. \tag{1}$$

   Solution. We make the substitution

   $$u = x^3+3x. \tag{2}$$

   Then

   $$du = 3(x^2+1)dx, \tag{3}$$

   so that

   $$\begin{aligned}\int (x^2+1)(x^3+3x)^4\,dx &= \frac{1}{3}\int u^4\,du, && (4)\\ &= \frac{1}{15}u^5 + C && (5)\\ &= \frac{1}{15}(x^3+3x)^5 + C. && (6)\end{aligned}$$

   □

   (b) Problem 48: Compute the indefinite integral

   $$\int x^3\sqrt{x^2+1}\,dx. \tag{7}$$

   Solution. We make the substitution

   $$u = x^2+1. \tag{8}$$

   Then

   $$\begin{aligned}du &= 2xdx, && (9)\\ x^2 &= u-1, && (10)\end{aligned}$$

so that

$$\int x^3\sqrt{x^2+1}\,dx = \frac{1}{2}\int (u-1)\sqrt{u}\,du, \tag{11}$$

$$= \frac{1}{2}\int u^{3/2} - u^{1/2}\,du \tag{12}$$

$$= \frac{1}{2}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C \tag{13}$$

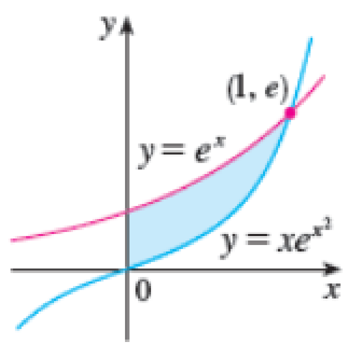
$$= \frac{1}{2}\left(\frac{2}{5}(x^2+1)^{5/2} - \frac{2}{3}(x^2+1)^{3/2}\right) + C. \tag{14}$$

□

2. Section 6.1 Problems

   (a) Problem 2: Find the area bounded by $y = e^x$, $y = xe^{x^2}$, and $x = 0$.

2.



Solution. The area A is computed by

$$A = \int_0^1 e^x - xe^{x^2}\,dx \tag{15}$$

$$= e - 1 - \int_0^1 xe^{x^2}\,dx. \tag{16}$$

We make the substitution

$$u = x^2, \tag{17}$$

so that

$$A = e - 1 - \frac{1}{2}\int_0^1 e^u\,du \tag{18}$$

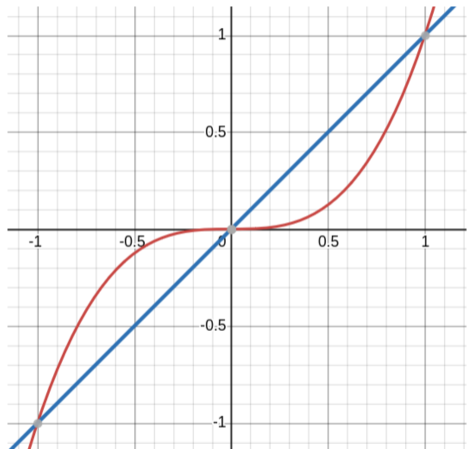
$$= e - 1 - \frac{1}{2}(e-1) \tag{19}$$

$$= \frac{e-1}{2}. \tag{20}$$

□

(b) Problem 22: Find the area bounded by $y = x^3$ and $y = x$.

Solution. Sketching the two curves gives us



Therefore, the area A is computed as

$$A = \int_{-1}^{0} x^3 - x\,dx + \int_{0}^{1} x - x^3\,dx \tag{21}$$
$$= 2\int_{0}^{1} x - x^3\,dx \tag{22}$$
$$= 2\left(\frac{1}{2} - \frac{1}{4}\right) \tag{23}$$
$$= \frac{1}{2}. \tag{24}$$

□