

7.3 Exercises

1, 2 and 3 Evaluate the integral using the indicated trigonometric substitution. Sketch and label the associated right triangle.

$$1. \int \frac{dx}{x^2 \sqrt{4-x^2}} \quad x = 2 \sin \theta$$

$$2. \int \frac{x^3}{\sqrt{x^2+4}} dx \quad x = 2 \tan \theta$$

$$3. \int \frac{\sqrt{x^2-4}}{x} dx \quad x = 2 \sec \theta$$

4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29 and 30 Evaluate the integral.

$$4. \int \frac{x^2}{\sqrt{9-x^2}} dx$$

$$5. \int \frac{\sqrt{x^2-1}}{x^4} dx$$

$$6. \int_0^3 \frac{x}{\sqrt{36-x^2}} dx$$

$$7. \int_0^a \frac{dx}{(a^2+x^2)^{3/2}}, \quad a > 0$$

$$8. \int \frac{dt}{t^2 \sqrt{t^2-16}}$$

$$9. \int_2^3 \frac{dx}{(x^2-1)^{3/2}}$$

$$10. \int_0^{2/3} \sqrt{4-9x^2} dx$$

$$11. \int_0^{1/2} x \sqrt{1-4x^2} dx$$

12. $\int_0^2 \frac{dt}{\sqrt{4+t^2}}$
13. $\int \frac{\sqrt{x^2-9}}{x^3} dx$
14. $\int_0^1 \frac{dx}{(x^2+1)^2}$
15. $\int_0^a x^2 \sqrt{a^2-x^2} dx$
16. $\int_{\sqrt{2}/3}^{2/3} \frac{dx}{x^5 \sqrt{9x^2-1}}$
17. $\int \frac{x}{\sqrt{x^2-7}} dx$
18. $\int \frac{dx}{[(ax)^2-b^2]^{3/2}}$
19. $\int \frac{\sqrt{1+x^2}}{x} dx$
20. $\int \frac{x}{\sqrt{1+x^2}} dx$
21. $\int_0^{0.6} \frac{x^2}{\sqrt{9-25x^2}} dx$
22. $\int_0^1 \sqrt{x^2+1} dx$
23. $\int \frac{dx}{\sqrt{x^2+2x+5}}$
24. $\int_0^1 \sqrt{x-x^2} dx$
25. $\int x^2 \sqrt{3+2x-x^2} dx$
26. $\int \frac{x^2}{(3+4x-4x^2)^{3/2}} dx$
27. $\int \sqrt{x^2+2x} dx$
28. $\int \frac{x^2+1}{(x^2-2x+2)^2} dx$

29.
$$\int x\sqrt{1-x^4} dx$$

30.
$$\int_0^{\pi/2} \frac{\cos t}{\sqrt{1+\sin^2 t}} dt$$

31.

a. Use trigonometric substitution to show that

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x + \sqrt{x^2+a^2}) + C$$

b. Use the hyperbolic substitution $x = a \sinh t$ to show that

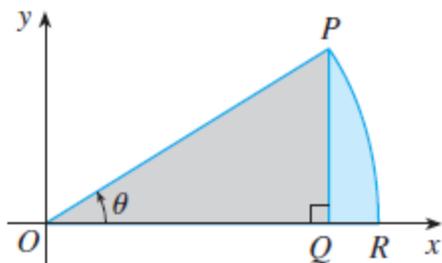
$$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

These formulas are connected by [Formula 3.11.3](#).

32. Evaluate

$$\int \frac{x^2}{(x^2+a^2)^{3/2}} dx$$

a. by trigonometric substitution.

b. by the hyperbolic substitution $x = a \sinh t$.33. Find the average value of $f(x) = \sqrt{x^2-1}/x$, $1 \leq x \leq 7$.34. Find the area of the region bounded by the hyperbola $9x^2 - 4y^2 = 36$ and the line $x = 3$.35. Prove the formula $A = \frac{1}{2}r^2\theta$ for the area of a sector of a circle with radius r and central angle θ . [Hint: Assume $0 < \theta < \pi/2$ and place the center of the circle at the origin so it has the equation $x^2 + y^2 = r^2$. Then A is the sum of the area of the triangle POQ and the area of the region PQR in the figure.]

36.  Evaluate the integral

$$\int \frac{dx}{x^4 \sqrt{x^2 - 2}}$$

Graph the integrand and its indefinite integral on the same screen and check that your answer is reasonable.

37. Find the volume of the solid obtained by rotating about the x -axis the region enclosed by the curves $y = 9/(x^2 + 9)$, $y = 0$, $x = 0$, and $x = 3$.

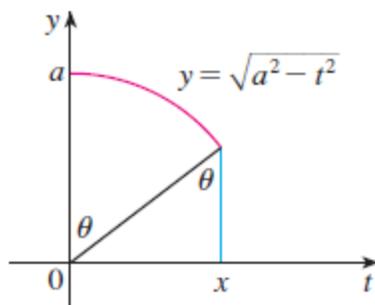
38. Find the volume of the solid obtained by rotating about the line $x = 1$ the region under the curve $y = x\sqrt{1 - x^2}$, $0 \leq x \leq 1$.

39.

- a. Use trigonometric substitution to verify that

$$\int_a^x \sqrt{a^2 - t^2} dt = \frac{1}{2}a^2 \sin^{-1}(x/a) + \frac{1}{2}x\sqrt{a^2 - x^2}$$

- b. Use the figure to give trigonometric interpretations of both terms on the right side of the equation in part (a).



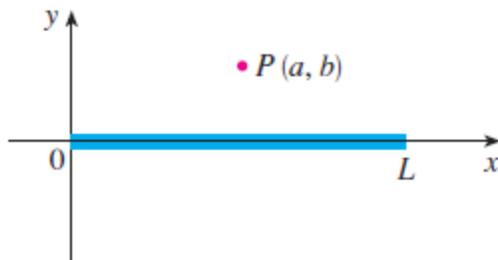
40. The parabola $y = \frac{1}{2}x^2$ divides the disk $x^2 + y^2 \leq 8$ into two parts. Find the areas of both parts.

41. A torus is generated by rotating the circle $x^2 + (y - R)^2 = r^2$ about the x -axis. Find the volume enclosed by the torus.

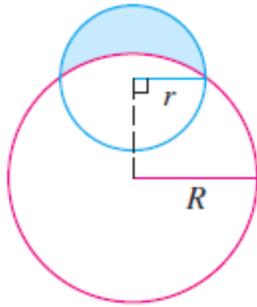
42. A charged rod of length L produces an electric field at point $P(a, b)$ given by

$$E(P) = \int_{-a}^{L-a} \frac{\lambda b}{4\pi\epsilon_0(x^2 + b^2)^{3/2}} dx$$

where λ is the charge density per unit length on the rod and ϵ_0 is the free space permittivity (see the [figure](#)). Evaluate the integral to determine an expression for the electric field $E(P)$.



43. Find the area of the crescent-shaped region (called a *lune*) bounded by arcs of circles with radii r and R . (See the figure.)



44. A water storage tank has the shape of a cylinder with diameter **10** ft. It is mounted so that the circular cross-sections are vertical. If the depth of the water is **7** ft, what percentage of the total capacity is being used?

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 Printed By: Jordan Hoffart (jordanhoffart@tamu.edu)
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