A Few Facts from Calculus (part 1)

1. For any integrable function $u : [a,b] \to \mathbb{R}$

$$\left| \int_a^b u(x)\,dx \right| \leq \int_a^b |u(x)|\,dx$$

2. If $u : [a,b] \to \mathbb{R}$ is continuously differentiable, then for any $a \leq x \leq y \leq b$,

$$\int_x^y u'(s)\,ds = u(y) - u(x)$$

3. Let $f, g : [a,b] \to \mathbb{R}$ be differentiable at $a \leq x \leq b$. Then $fg : [a,b] \to \mathbb{R}$ is differentiable at x and

$$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$$

4. If $f(x) \leq g(x)$ for all $a \leq x \leq b$, then

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx .$$

5. For any integrable $f, g : [a,b] \to \mathbb{R}$

$$\left| \int_a^b f(x)g(x)\,dx \right| \le \left(\int_a^b f(x)^2 dx \right)^{1/2} \left(\int_a^b g(x)^2 dx \right)^{1/2}$$

6. If $u : [a,b] \to \mathbb{R}$ has a local minimum at some $a < x_0 < b$, then $u'(x_0) = 0$.

The same is true if we replace "minimum" by "maximum". Warning: the strict inequalities $a < x_0 < b$ are essential; the theorem could fail if $a = x_0$ or $b = x_0$.