Example for How (Not) to write a proof

Proposition For any real numbers a and b,

$$ab \leq \frac{1}{2}(a^2 + b^2).$$

Good proof. Let a, b be two real numbers.

Then $(a-b)^2 = a^2 - 2ab + b^2$ and $(a-b)^2 \geq 0$.

Therefore, $a^2 - 2ab + b^2 \geq 0$. Adding $2ab$ to both sides gives us

$$a^2 + b^2 \geq 2ab,$$

which is equivalent to what we want to show. □

Bad proof.

$$(a-b)^2 = a^2 - 2ab + b^2 \geq 0$$

$$a^2 + b^2 \geq 2ab \quad \square$$

The good proof is good because (1) it is written in complete, (mostly) grammatically correct English,

(2) each step of the proof logically follows from the previous step, and

(3) each step is reasonably explained and justified.

The bad proof is bad because

(1) the assumptions are not stated

(2) No mathematical expressions are explained

(3) there is no coherent argument connecting each mathematical expression

(4) There is no flow of logic.