Math 152 – Python Lab 4

Directions: Use Python to solve each problem, unless the question states otherwise. For this lab, approximate answers (that is, decimal versions of numbers) are acceptable for all non-plotting questions. (Template link)

1. Given

$$f(x) = \frac{x^2}{e^x - 1},$$

   (a) Plot the function with x-interval $[0, 10]$. (Note this is different from the integral bounds you'll be using for the rest of the problem!)

   (b) Recall the **Left Endpoint Riemann Sum** from MATH 151:

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} f(x_{i-1})\Delta x,$$

   wherre $\Delta x = \frac{b-a}{n}$ and $x_i = a + i \cdot \Delta x$. We can compute this in Python with the following steps:

   (i) Define a list x from a (inclusive) to b (exclusive) with step size Δx.

   (ii) Define a list $y = f(x)$.

   (iii) Sum the list y and multiply by Δx.

   Compute the Left endpoint approximation of $\int_1^5 f(x)dx$ using $n = 500$ subintervals.

2. The **Right Endpoint Riemann Sum** is the same process, but starting at $a + \Delta x$ and ending at b (inclusive).

   Compute the Right endpoint approximation of $\int_1^5 f(x)dx$ using $n = 500$ subintervals.

3. The **Midpoint Riemann Sum** is again the same process, but starting at $a + \frac{\Delta x}{2}$ and ending at $b - \frac{\Delta x}{2}$ (inclusive).

   (a) Compute the Midpoint approximation of $\int_1^5 f(x)dx$ using $n = 500$ subintervals.

   (b) Compute the average of the Left and Right Endpoint approximations. Is this equal to the Midpoint approximation?

   (There are problems on the back!)

4. Another approximation for integrals is the Trapezoid Rule:
$$\int_a^b f(x)dx \approx \frac{\Delta x}{2}\big(f(x_0) + 2f(x_1) + 2f(x_2) + ...2f(x_{n-1}) + f(x_n)\big).$$

   There is a built-in function **trapezoid** in the package **scipy.integrate** (see the overview for more information).

   (a) Compute the Trapezoid approximation using $n = 500$ subintervals.

   (b) Is the Trapezoid approximation equal to the average of the Left and Right Endpoint approximations?

5. Simpson's Method is another approximation to the integral which uses parabolas instead of lines to approximate the integral:
$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}\big(f(x_0) + 2f(x_1) + 4f(x_2) + ... + 4f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\big)$$

   Compute the Simpson approximation using $n = 500$ subintervals using the **simpson** command from the **scipy.integrate** package (again, see the overview).

6. The value of the integral to 15 decimal places is 1.800175254562853. Use this value to estimate the error |actual − estimate| in each of the five approximations.