

## Math 152 - Python Lab 5

**Directions**: Use Python to solve each problem, unless the question states otherwise. (Template link)

1. Given the integral

$$\int \frac{x^3 + x^2 - 12x + 1}{x^2 + x - 12} dx,$$

(a) Use **sp.div** to break up the above fraction via long division. Here's the syntax:

## q, r = sp.div(numerator, denominator)

This gives q, the quotient, and r, the remainder, from the division.

Display the reduced function (it should have the form  $q + \frac{r}{\text{denominator}}$ ).

- (b) Write the form of the partial fraction decomposition for the remaining fraction (by hand). Use it to set up and solve a system of equations (in Python) to obtain the missing coefficients A, B, etc.
- (c) Check your answer to part (B) by using the **sp.apart** command.
- (d) Use your decomposition from Parts (A) and (B) or use the output from Part (C) to evaluate the integral. Check your answer by integrating directly.
- 2. (a) Calculate the value of a so that  $\int_0^\infty \frac{x^4}{x^7 + a^2} dx = 0.1$ . (Note: When defining x and a as symbolic variables, include **positive = True** to clear up some issues when solving.)
  - (b) Find the value of a such that  $\int_2^a x^6 e^{-x^7} dx = \int_a^\infty x^6 e^{-x^7} dx.$

(c) Evaluate 
$$\int_{2}^{\infty} x^{6} e^{-x^{7}} dx$$
 using the value found in part (b).

- 3. Let  $f(x) = \frac{\sin^2(x) + 2}{x}$  and  $g(x) = \frac{1}{x}$ .
  - (a) Show  $\int_{1}^{\infty} g(x) dx$  diverges using Python.
  - (b) Plot f and g on the same axes with domain [1,40] to show  $f(x) \ge g(x)$  on the given interval.
  - (c) Show  $\int_{1}^{\infty} f(x)dx$  diverges using Python.

(d) Using what you know from parts (a) and (b), how could you conclude that  $\int_{1}^{\infty} f(x)dx$  diverges WITHOUT using Python to evaluate the integral?