Math 152 – Python Lab 8

Directions: Use Python to solve each problem, unless the question states otherwise. (Template link)

1. Given the series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{12n+100}$:

   (a) Using the Remainder Estimate for the Alternating Series Test for N terms, plot the upper bound (function) in the window and the line $y = 0.001$. Use your graph to determine how many terms are needed to sum the series to within 0.001.

   (b) Use **sp.nsolve** to confirm your graphical answer from part (a).

   (c) Find the sum of the series to within 0.001.

   (d) Use Python to determine whether the series is absolutely convergent.

2. Given $\sum_{n=0}^{\infty} \frac{(2n)!}{(5000)^n}$:

   (a) Print the first 16 terms of the series (from a_0 to a_{15}). Based on your output, do you expect the series to converge or diverge?

   (b) Apply the Ratio Test to the series, i.e., compute $\left|\frac{a_{n+1}}{a_n}\right|$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$.

   (c) What does your answer to part (b) tell you about the series?

3. Given the power series $\sum_{n=0}^{\infty}(-1)^n \frac{x^{8n+4}}{6n+3}$:

   (a) Simplify $\left|\frac{a_{n+1}}{a_n}\right|$ and compute the limit $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$.

   (b) State the interval of convergence for this power series. Remember to check the endpoints, if necessary.

   (c) Find the partial sums s_1, s_3, and s_5 for this power series. You can use the **sp.summation** command for this.

   (d) It can be shown that the series converges to $f(x) = \frac{1}{3}\arctan(x^4)$. Plot s_1, s_3, s_5, and f on the same graph to illustrate this. Use the interval of convergence found in part (b) as your plot's x-domain. If the interval is a single x-value or $(-\infty, \infty)$, use x-domain $[-1.5, 1.5]$ instead.