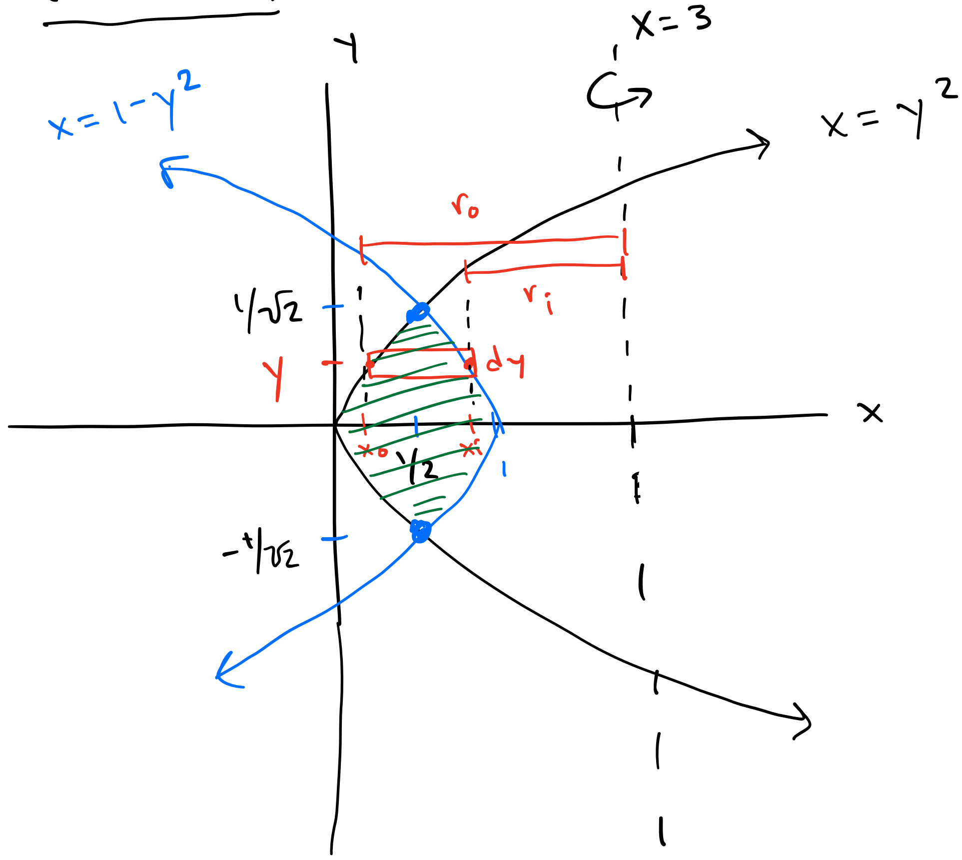
6.2 (17)

$1-y^2 = y^2$ when $y = \pm \frac{1}{\sqrt{2}}$



when $y = \pm 1/\sqrt{2}$ and $x = y^2$, $x = \frac{1}{2}$

At height y between $-1/\sqrt{2}$ and $1/\sqrt{2}$, we have a cross section that is a washer of inner radius r_i, outer radius r_o, and thickness dy.

The inner radius r_i is the distance from the line $x = 3$ to the point x_i, where (x_i, y) lies on the curve $x_i = 1 - y^2$. This distance is therefore $r_i = 3 - (1-y^2)$. Similarly, the outer radius $r_o = 3 - x_o = 3 - y^2$.

Thus, the area of the washer at height y

is $A(y) = \pi \left(r_o^2 - r_i^2 \right)$

$$= \pi \left((3-y^2)^2 - (3-(1-y^2))^2 \right).$$

So the volume given by rotating the green area around the line $x = 3$ is

$$V = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} A(y)\, dy = \pi \int_{-1/\sqrt{2}}^{1/\sqrt{2}} (3-y^2)^2 - (2+y^2)^2\, dy$$

$$= \pi \int_{-1/\sqrt{2}}^{1/\sqrt{2}} 9 - 6y^2 + y^4 - (4 + 4y^2 + y^4)\, dy$$

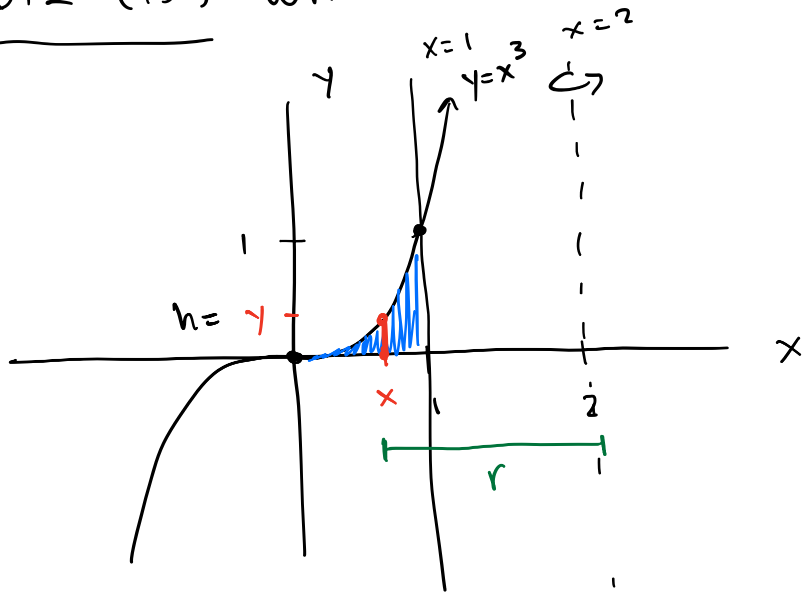
$$= \pi \int_{-1/\sqrt{2}}^{1/\sqrt{2}} 5 - 10y^2\, dy = 5\pi \int_{-1/\sqrt{2}}^{1/\sqrt{2}} 1 - 2y^2\, dy$$

$$= 10\pi \int_0^{1/\sqrt{2}} 1 - 2y^2\, dy = 10\pi \left(y - \frac{2}{3} y^3 \right) \Big|_0^{1/\sqrt{2}}$$

$$= 10\pi \left(\frac{1}{\sqrt{2}} - 2/3 \frac{1}{(\sqrt{2})^3} \right)$$

$$= \frac{10\pi}{\sqrt{2}} \left(1 - \frac{1}{3} \right) = \frac{20\pi}{3\sqrt{2}} = \frac{10\sqrt{2}}{3} \pi \quad \square$$

<u>6.2 (15)</u> with shell



At $0 \le x \le 1$, we have a thin cylindrical shell of radius $r = 2-x$ and height $h = y = x^3$.

Its area is then

$$A(x) = 2\pi r h = 2\pi(2-x)x^3$$

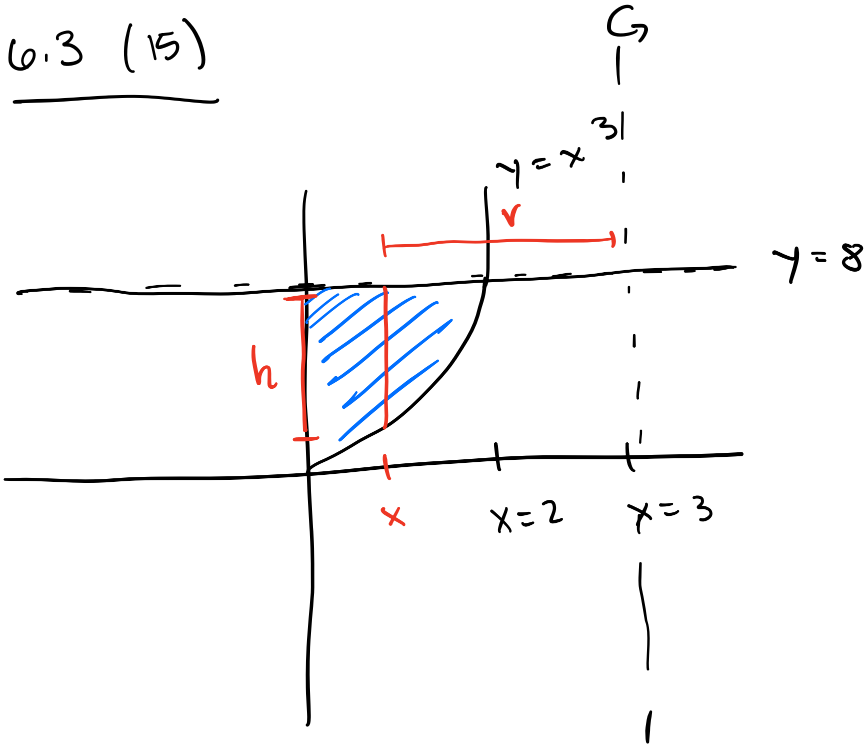
Therefore, the volume obtained by rotating the blue region about the line $x = 2$ is

$$V = \int_0^1 A(x)\,dx = \int_0^1 2\pi(2-x)x^3\,dx$$

$$= 2\pi \int_0^1 2x^3 - x^4 = 2\pi\left(\tfrac{1}{2}x^4 - \tfrac{1}{5}x^5\right)\Big|_0^1$$

$$= 2\pi\left(1/2 - 1/5\right) = \frac{6\pi}{10} = \frac{3\pi}{5} \qquad \square$$

6.3 (15)



$$h = 8 - x^3$$

$$r = 3 - x$$

$$A(x) = 2\pi r h = 2\pi(3-x)(8-x^3)$$

$$V = \int_0^2 A(x)\,dx = 2\pi\int_0^2 (3-x)(8-x^3)\,dx$$

$$= \ldots \quad \text{(exercise!)}$$