Chapter 5: Integrals: 5.3 Exercises
Book Title: Calculus: Early Transcendentals
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5.3 Exercises

1. Explain exactly what is meant by the statement that "differentiation and integration are inverse processes."

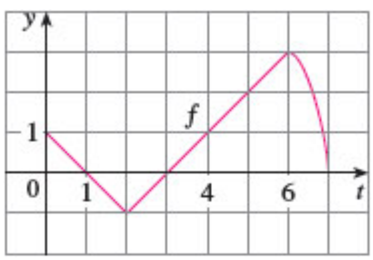
2. Let $g(x) = \int_0^x f(t)\, dt$, where f is the function whose graph is shown.

   a. Evaluate $g(x)$ for $x = 0, 1, 2, 3, 4, 5$, and 6.

   b. Estimate $g(7)$.

   c. Where does g have a maximum value? Where does it have a minimum value?

   d. Sketch a rough graph of g.



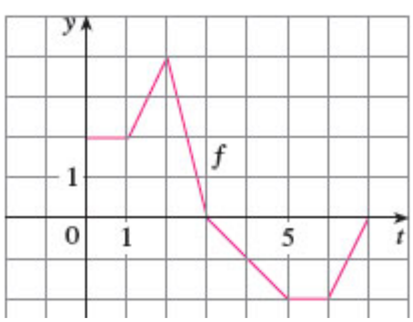
3. Let $g(x) = \int_0^x f(t)\, dt$, where f is the function whose graph is shown.

   a. Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$, and $g(6)$.

   b. On what interval is g increasing?

   c. Where does g have a maximum value?

   d. Sketch a rough graph of g.



4. Let $g(x) = \int_0^x f(t)\, dt$, where f the function whose graph is shown.

a. Evaluate $g(0)$ and $g(6)$.

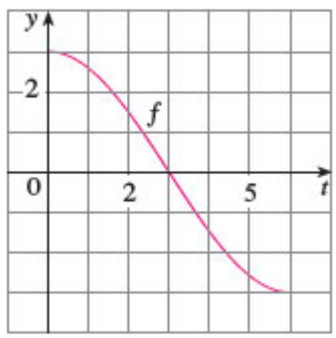
b. Estimate $g(x)$ for $x = 1, 2, 3, 4,$ and 5.

c. On what interval is g increasing?

d. Where does g have a maximum value?

e. Sketch a rough graph of g.

f. Use the graph in part (e) to sketch the graph of $g'(x)$. Compare with the graph of f.



5 and 6 Sketch the area represented by $g(x)$. Then find $g'(x)$ in two ways:

(a) by using Part 1 of the Fundamental Theorem and

(b) by evaluating the integral using Part 2 and then differentiating.

5. $g(x) = \int_1^x t^2\, dt$

6. $g(x) = \int_0^x (2 + \sin t)\, dt$

7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17 and 18 Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

7. $g(x) = \int_0^x \sqrt{t + t^3}\, dt$

8. $g(x) = \int_1^x \ln\left(1 + t^2\right)\, dt$

9. $g(s) = \int_5^s (t - t^2)^8 \, dt$

10. $h(u) = \int_0^u \frac{\sqrt{t}}{t+1} \, dt$

11. $F(x) = \int_x^0 \sqrt{1 + \sec t} \, dt$

$\left[\textit{Hint:} \int_x^0 \sqrt{1 + \sec t} \, dt = -\int_0^x \sqrt{1 + \sec t} \, dt \right]$

12. $R(y) = \int_y^2 t^3 \sin t \, dt$

13. $h(x) = \int_1^{e^x} \ln t \, dt$

14. $h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4 + 1} dz$

15. $y = \int_1^{3x+2} \frac{t}{1 + t^3} \, dt$

16. $y = \int_0^{x4} \cos^2 \theta \, d\,\theta$

17. $y = \int_{\sqrt{x}}^{\pi/4} \theta \tan \theta \, d\,\theta$

18. $y = \int_{\sin x}^1 \sqrt{1 + t^2} \, dt$

19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43 and 44 Evaluate the integral.

19. $\int_1^3 (x^2 + 2x - 4) \, dx$

20. $\int_{-1}^1 x^{100} \, dx$

21. $\int_0^2 \left(\frac{4}{5}t^3 - \frac{3}{4}t^2 + \frac{2}{5}t \right) \, dt$

22. $\int_0^1 (1 - 8v^3 + 16v^7) \, dv$

23. $\int_1^9 \sqrt{x} \, dx$

24. $\int_{1}^{8} x^{-2/3}\ dx$

25. $\int_{\pi/6}^{\pi} \sin\theta\ d\theta$

26. $\int_{-5}^{5} e\ dx$

27. $\int_{0}^{1} (u+2)(u-3)\ du$

28. $\int_{0}^{4} (4-t)\sqrt{t}\ dt$

29. $\int_{1}^{4} \frac{2+x^2}{\sqrt{x}}\ dx$

30. $\int_{-1}^{2} (3u-2)\ (u+1)\ du$

31. $\int_{\pi/6}^{\pi/2} \csc t \cot t\, dt$

32. $\int_{\pi/4}^{\pi/3} \csc^2\theta\ d\theta$

33. $\int_{0}^{1} (1+r)^3\ dr$

34. $\int_{0}^{3} (2\sin x - e^x)\ dx$

35. $\int_{1}^{2} \frac{v^3+3v^6}{v^4}\ dv$

36. $\int_{1}^{18} \sqrt{\frac{3}{z}}\ dz$

37. $\int_{0}^{1} (x^e + e^x)\ dx$

38. $\int_{0}^{1} \cosh t\ dt$

39. $\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1+x^2} dx$

40. $\int_{1}^{3} \frac{y^3-2y^2-y}{y^2} dy$

41. $\int_{0}^{4} 2^{s}\, ds$

42. $\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^{2}}} dx$

43. $\int_{0}^{\pi} f(x)\, dx$ where $f(x) = \begin{cases} \sin x \text{ if } 0 \leqslant x < \pi/2 \\ \cos x \text{ if } \pi/2 \leqslant x \leqslant \pi \end{cases}$

44. $\int_{-2}^{2} f(x)\, dx$ where $f(x) = \begin{cases} 2 & \text{if } -2 \leqslant x \leqslant 0 \\ 4 - x^{2} & \text{if } 0 < x \leqslant 2 \end{cases}$

45, 46, 47 and 48 Sketch the region enclosed by the given curves and calculate its area.

45. $y = \sqrt{x}, \quad y = 0, \quad x = 4$

46. $y = x^{3}, \quad y = 0, \quad x = 1$

47. $y = 4 - x^{2}, \quad y = 0$

48. $y = 2x - x^{2}, \quad y = 0$

49, 50, 51 and 52 Use a graph to give a rough estimate of the area of the region that lies beneath the given curve. Then find the exact area.

49. $y = \sqrt[3]{x}, \quad 0 \leqslant x \leqslant 27$

50. $y = x^{-4}, \quad 1 \leqslant x \leqslant 6$

51. $y = \sin x, \ 0 \leqslant x \leqslant \pi$

52. $y = \sec^{2} x, \ 0 \leqslant x \leqslant \pi/3$

53 and 54 evaluate the integral and interpret it as a difference of areas. Illustrate with a sketch.

53. $\int_{-1}^{2} x^{3}\, dx$

54.

$$\int_{\pi/6}^{2\pi} \cos x \, dx$$

55, 56, 57 and 58 what is wrong with the equation?

55. $\int_{-2}^{1} x^{-4} dx = \left.\frac{x^{-3}}{-3}\right]_{-2}^{1} = -\frac{3}{8}$

56. $\int_{-1}^{2} \frac{4}{x^3} \, dx = -\left.\frac{2}{x^2}\right]_{-1}^{2} = \frac{3}{2}$

57. $\int_{\pi/3}^{\pi} \sec \theta \tan \theta \, d\theta = \sec \theta]_{\pi/3}^{\pi} = -3$

58. $\int_{0}^{\pi} \sec^2 x \, dx = \tan x]_0^{\pi} = 0$

59, 60, 61, 62 and 63 find the derivative of the function.

59. $g(x) = \int_{2x}^{3x} \frac{u^2 - 1}{u^2 + 1} \, du$

$\left[\text{Hint: } \int_{2x}^{3x} f(u) \, du = \int_{2x}^{0} f(u) \, du + \int_{0}^{3x} f(u) \, du\right]$

60. $g(x) = \int_{1-2x}^{1+2x} t \sin t \, dt$

61. $F(x) = \int_{x}^{x^2} e^{t^2} \, dt$

62. $F(x) = \int_{\sqrt{x}}^{2x} \arctan t \, dt$

63. $y = \int_{\cos x}^{\sin x} \ln(1 + 2v) \, dv$

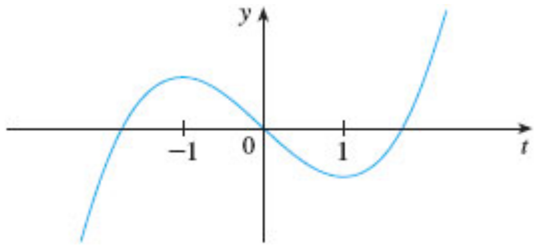
64. If $f(x) = \int_{0}^{x} \left(1 - t^2\right) e^{t^2} \, dt$, on what interval is f increasing?

65. On what interval is the curve?

$$y = \int_{0}^{x} \frac{t^2}{t^2 + t + 2} dt$$

Concave downward?

66. Let $F(x) = \int_1^x f(t)\, dt$, where f is the function whose graph is shown. Where is F concave downward?



67. Let $F(x) = \int_2^x e^{t^2}\, dt$. Find an equation of the tangent line to the curve $y = F(x)$ at the point with x-coordinate 2.

68. If $f(x) = \int_0^{\sin x} \sqrt{1+t^2}\, dt$ any $g(y) = \int_3^y f(x)\, dx$, find $g''(\pi/6)$.

69. If $f(1) = 12$, f' is continuous, and $\int_1^4 f'(x)\, dx = 17$, what is the value of $f(4)$?

70. The **error function**

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

is used in probability, statistics, and engineering.

a. Show that $\int_a^b e^{-t^2}\, dt = \frac{1}{2}\sqrt{\pi}\,[\operatorname{erf}(b) - \operatorname{erf}(a)]$.

b. Show that the function $y = e^{x^2}\operatorname{erf}(x)$ satisfies the differential equation $y' = 2xy + 2/\sqrt{\pi}$.

71. The Fresnel function S was defined in Example 3 and graphed in Figures 7 and 8.

a. At what values of x does this function have local maximum values?

b. On what intervals is the function concave upward?

c. CAS Use a graph to solve the following equation correct to two decimal places:

$$\int_0^x \sin(\pi t^2/2)\, dt = 0.2$$

72. CAS The **sine integral function**

$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t} dt$$

is important in electrical engineering. [The integrand $f(t) = (\sin t)/t$ is not defined when $t = 0$, but we know that its limit is 1 when $t \to 0$. So we define $f(0) = 1$ and this makes f a continuous function everywhere.]

a. Draw the graph of Si.

b. At what values of x does this function have local maximum values?

c. Find the coordinates of the first inflection point to the right of the origin.

d. Does this function have horizontal asymptotes?

e. Solve the following equation correct to one decimal place:

$$\int_0^x \frac{\sin t}{t} dt = 1$$

73 and 74 let $g(x) = \int_0^x f(t)\, dt$, where f the function whose graph is shown.

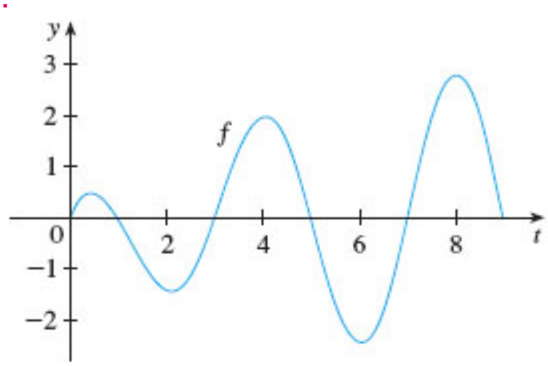
a. At what values of x do the local maximum and minimum values of g occur?

b. Where does g attain its absolute maximum value?

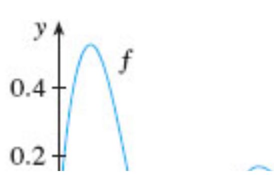
c. On what intervals is g concave downward?

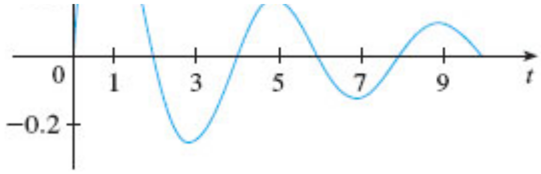
d. Sketch the graph of g.

73.



74.





75 and 76 Evaluate the limit by first recognizing the sum as a Riemann sum for a function defined on $[0, 1]$.

75. $\lim_{n\to\infty} \sum_{i-1}^{n} \left(\frac{i^4}{n^5} + \frac{i}{n^2}\right)$

76. $\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \sqrt{\frac{3}{n}} + \cdots + \sqrt{\frac{n}{n}}\right)$

77. Justify (3) for the case $h < 0$.

78. If f is continuous and g and h are differentiable functions, find a formula for

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\, dt$$

79.

a. Show that $1 \leqslant \sqrt{1+x^3} \leqslant 1+x^3$ for $x \geqslant 0$.

b. Show $1 \leqslant \int_0^1 \sqrt{1+x^3}\, dx \leqslant 1.25$.

80.

a. Show that $\cos(x^2) \geqslant \cos x$ for $0 \leqslant x \leqslant 1$.

b. Deduce that $\int_0^{\pi/6} \cos(x^2)\, dx \geqslant \frac{1}{2}$.

81. Show that

$$0 \leqslant \int_5^{10} \frac{x^2}{x^4+x^2+1} dx \leqslant 0.1$$

by comparing the integrand to a simpler function.

82. Let

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leqslant x \leqslant 1 \\ 2 - x & \text{if } 1 < x \leqslant 2 \\ 0 & \text{if } x > 2 \end{cases}$$

and

$$g(x) = \int_0^x f(t)\,dt$$

a. Find an expression for $g(x)$ similar to the one $f(x)$.

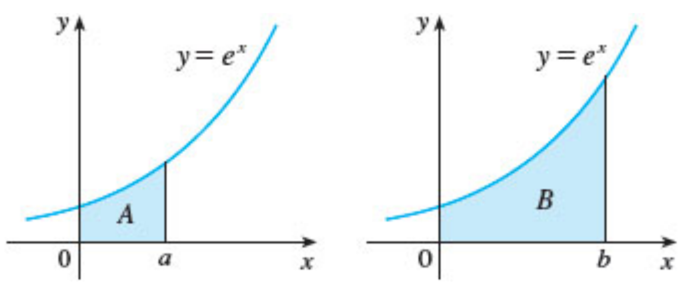
b. Sketch the graphs of f and g.

c. Where is f differentiable? Where is g differentiable?

83. Find a function f and a number a such that

$$6 + \int_a^x \frac{f(t)}{t^2} dt = 2\sqrt{x} \qquad \text{for all } x > 0$$

84. The area labeled B is three times the area A. Express b in terms of a.



85. A manufacturing company owns a major piece of equipment that depreciates at the (continuous) rate $f = f(t)$, where t is the time measured in months since its last overhaul. Because a fixed cost A is incurred each time the machine is overhauled, the company wants to determine the optimal time T (in months) between overhauls.

a. Explain why $\int_0^t f(s)\,ds$ represents the loss in value of the machine over the period of time t since the last overhaul.

b. Let $C = C(t)$ be given by

$$C(t) = \frac{1}{t}\left[A + \int_0^t f(s)\,ds\right]$$

What does C represent and why would the company want to minimize C?

c. Show that C has a minimum value at the numbers $t = T$ where $C(T) = f(T)$.

86. A high-tech company purchases a new computing system whose initial value is V. The system will depreciate at the rate $f = f(t)$ and will accumulate maintenance costs at the rate $g = g(t)$, where t is the time measured in months. The company wants to determine the optimal time to replace the system.

a. Let

$$C(t) = \frac{1}{t} \int_0^t [f(s) + g(s)]\, ds$$

Show that the critical numbers of C occur at the numbers t where $c(t) = f(t) + g(t)$.

b. Suppose that

$$f(t) = \begin{cases} \frac{V}{15} - \frac{V}{450}t & \text{if } t > 30 \\ 0 & \text{if } t > 30 \end{cases}$$

and

$$g(t) = \frac{Vt^2}{12{,}900} \qquad t > 0$$

Determine the length of time T for the total depreciation $D(t) = \int_0^t f(s)\, ds$ to equal the initial value V.

c. Determine the absolute minimum of C $(0, T]$.

d. Sketch the graphs of C and $f + g$ in the same coordinate system, and verify the result in part (a) in this case.

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