Chapter 5: Integrals: 5.5 Exercises
Book Title: Calculus: Early Transcendentals
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5.5 Exercises

1, 2, 3, 4, 5 and 6 Evaluate the integral by making the given substitution.

1. $\int \cos 2x \, dx, \quad u = 2x$

2. $\int xe^{-x^2} \, dx, \quad u = -x^2$

3. $\int x^2 \sqrt{x^3 + 1} \, dx, \quad u = x^3 + 1$

4. $\int \sin^2 \theta \cos \theta \, d\theta, \quad u = \sin \theta$

5. $\int \frac{x^3}{x^4 - 5} \, dx, \quad u = x^4 - 5$

6. $\int \sqrt{2t + 1} \, dt, \quad u = 2t + 1$

7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47 and 48 Evaluate the indefinite integral.

7. $\int x\sqrt{1 - x^2} \, dx$

8. $\int x^2 e^{x^3} \, dx$

9. $\int (1 - 2x)^9 \, dx$

10. $\int \sin t \sqrt{1 + \cos t} \, dt$

11. $\int \cos (\pi t/2) \, dt$

12. $\int \sec^2 2\theta \, d\theta$

13. $\int \frac{dx}{5-3x}$

14. $\int y^2 \left(4-y^3\right)^{2/3}\, dy$

15. $\int \cos^3\theta \sin\theta \, d\theta$

16. $\int e^{-5r}\, dr$

17. $\int \frac{e^u}{(1-e^u)^2}\, du$

18. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

19. $\int \frac{a+bx^2}{\sqrt{3ax+bx^3}}\, dx$

20. $\int \frac{z^2}{z^3+1}\, dz$

21. $\int \frac{(\ln x)^2}{x}\, dx$

22. $\int \sin x \sin(\cos x)\, dx$

23. $\int \sec^2\theta \tan^3\theta \, d\theta$

24. $\int x\sqrt{x+2}\, dx$

25. $\int e^x\sqrt{1+e^x}\, dx$

26. $\int \frac{dx}{ax+b}\ (a \neq 0)$

27. $\int \left(x^2+1\right)\left(x^3+3x\right)^4 dx$

28. $\int e^{\cos t}\sin t\, dt$

29. $\int 5^t \sin\left(5^t\right) dt$

30. $\int \frac{\sec^2 x}{\tan^2 x}\, dx$

31. $\displaystyle\int \frac{(\arctan x)^2}{x^2+1}\,dx$

32. $\displaystyle\int \frac{x}{x^2+4}\,dx$

33. $\displaystyle\int \cos(1+5t)\,dt$

34. $\displaystyle\int \frac{\cos(\pi/x)}{x^2}\,dx$

35. $\displaystyle\int \sqrt{\cot x}\,\csc^2 x\,dx$

36. $\displaystyle\int \frac{2^t}{2^t+3}\,dt$

37. $\displaystyle\int \sinh^2 x\cosh x\,dx$

38. $\displaystyle\int \frac{dt}{\cos^2 t\sqrt{1+\tan t}}$

39. $\displaystyle\int \frac{\sin 2x}{1+\cos^2 x}\,dx$

40. $\displaystyle\int \frac{\sin x}{1+\cos^2 x}\,dx$

41. $\displaystyle\int \cot x\,dx$

42. $\displaystyle\int \frac{\cos(\ln t)}{t}\,dt$

43. $\displaystyle\int \frac{dx}{\sqrt{1-x^2}\,\sin^{-1} x}$

44. $\displaystyle\int \frac{x}{1+x^4}\,dx$

45. $\displaystyle\int \frac{1+x}{1+x^2}\,dx$

46. $\displaystyle\int x^2\sqrt{2+x}\,dx$

47. $\displaystyle\int x(2x+5)^8\,dx$

48. $\displaystyle\int x^3\sqrt{x^2+1}\,dx$

49, 50, 51 and 52 Evaluate the indefinite integral. Illustrate and check that your answer is reasonable by graphing both the function and its antiderivative (take $C = 0$).

49. $\int x\left(x^2-1\right)^3\,dx$

50. $\int \tan^2\theta\,\sec^2\theta\,d\theta$

51. $\int e^{\cos x}\sin x\,dx$

52. $\int \sin x\cos^4 x\,dx$

53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72 and 73 Evaluate the definite integral.

53. $\int_0^1 \cos(\pi t/2)\,dt$

54. $\int_0^1 (3t-1)^{50}\,dt$

55. $\int_0^1 \sqrt[3]{1+7x}\,dx$

56. $\int_0^3 \frac{dx}{5x+1}$

57. $\int_0^{\pi/6} \frac{\sin t}{\cos^2 t}dt$

58. $\int_{\pi/3}^{2\pi/3} \csc^2\left(\frac{1}{2}t\right)\,dt$

59. $\int_1^2 \frac{e^{1/x}}{x^2}\,dx$

60. $\int_0^1 xe^{-x^2}\,dx$

61. $\int_{-\pi/4}^{\pi/4} \left(x^3+x^4\tan x\right)\,dx$

62. $\int_0^{\pi/2} \cos x\sin(\sin x)\,dx$

63. $\displaystyle\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}}$

64. $\displaystyle\int_0^a x\sqrt{a^2 - x^2}\, dx$

65. $\displaystyle\int_0^a x\sqrt{x^2 + a^2}\, dx \ (a > 0)$

66. $\displaystyle\int_{-\pi/3}^{\pi/3} x^4 \sin x\, dx$

67. $\displaystyle\int_1^2 x\sqrt{x-1}\, dx$

68. $\displaystyle\int_0^4 \frac{x}{\sqrt{1+2x}}\, dx$

69. $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

70. $\displaystyle\int_0^2 (x-1)\, e^{(x-1)^2}\, dx$

71. $\displaystyle\int_0^1 \frac{e^z + 1}{e^z + z}\, dz$

72. $\displaystyle\int_0^{T/2} \sin(2\pi t/T - \alpha)\, dt$

73. $\displaystyle\int_0^1 \frac{dx}{(1+\sqrt{x})^4}$

74. Verify that $f(x) = \sin \sqrt[3]{x}$ is an odd function and use that fact to show that

$$0 \leqslant \int_{-2}^{3} \sin \sqrt[3]{x}\, dx \leqslant 1$$

75 and 76 Use a graph to give a rough estimate of the area of the region that lies under the given curve. Then find the exact area.

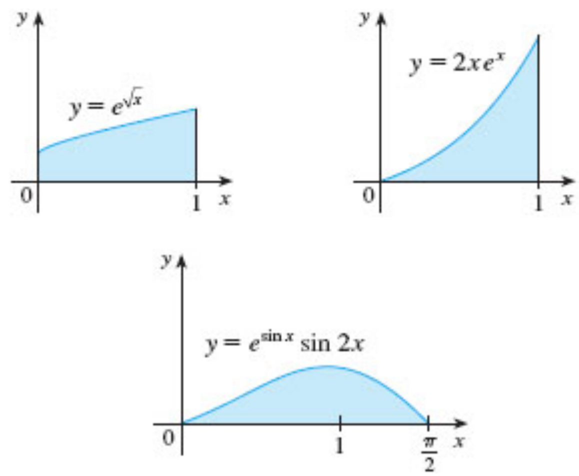
75. $y = \sqrt{2x+1}, \quad 0 \leqslant x \leqslant 1$

76. $y = 2\sin x - \sin 2x, \quad 0 \leqslant x \leqslant \pi$

77. Evaluate $\int_{-2}^{2} (x+3)\sqrt{4-x^2}\,dx$ by writing it as a sum of two integrals and interpreting one of those integrals in terms of an area.

78. Evaluate $\int_{0}^{1} x\sqrt{1-x^4}\,dx$ by making a substitution and interpreting the resulting integral in terms of an area.

79. Which of the following areas are equal? Why?



80. A model for the basal metabolism rate, in kcal/h, of a young man is $R(t) = 85 - 0.18\cos(\pi t/12)$, where t is the time in hours measured from 5:00 am. What is the total basal metabolism of this man, $\int_{0}^{24} R(t)\,dt$, over a 24-hour time period?

81. An oil storage tank ruptures at time $t = 0$ and oil leaks from the tank at a rate of $r(t) = 100e^{-0.01t}$ liters per minute. How much oil leaks out during the first hour?

82. A bacteria population starts with 400 bacteria and grows at a rate of $r(t) = (450.268)\,e^{1.12567t}$ bacteria per hour. How many bacteria will there be after three hours?

83. Breathing is cyclic and a full respiratory cycle from the beginning of inhalation to the end of exhalation takes about 5 s. The maximum rate of air flow into the lungs is about $0.5\ \mathrm{L/s}$. This explains, in part, why the function $f(t) = \frac{1}{2}\sin(2\pi t/5)$ has often been used to model the rate of air flow into the lungs. Use this model to find the volume of inhaled air in the lungs at time t.

84. The rate of growth of a fish population was modeled by the equation

$$G(t) = \frac{60{,}000e^{-0.6t}}{(1+5e^{-0.6t})^2}$$

where t is measured in years and G in kilograms per year. If the biomass was $25{,}000$ kg in the year 2000, what is the predicted biomass for the year 2020?

85. Dialysis treatment removes urea and other waste products from a patient's blood by diverting some of the bloodflow externally through a machine called a dialyzer. The rate at which urea is removed from the blood (in mg/min) is often well described by the equation

$$u(t) = \frac{r}{V}\, C_0 e^{-rt/V}$$

where r is the rate of flow of blood through the dialyzer (in mL/min), V is the volume of the patient's blood (in mL), and C_0 is the amount of urea in the blood (in mg) at time $t = 0$. Evaluate the integral $\int_0^{30} u(t)\, dt$ and interpret it.

86. Alabama Instruments Company has set up a production line to manufacture a new calculator. The rate of production of these calculators after t weeks is

$$\frac{dx}{dt} = 5000\left(1 - \frac{100}{(t+10)^2}\right) \text{calculators/week}$$

(Notice that production approaches 5000 per week as time goes on, but the initial production is lower because of the workers' unfamiliarity with the new techniques.) Find the number of calculators produced from the beginning of the third week to the end of the fourth week.

87. If f is continuous and $\int_0^4 f(x)\, dx = 10$, find $\int_0^2 f(2x)\, dx$.

88. If f is continuous and $\int_0^9 f(x)\, dx = 4$, find $\int_0^3 x f(x^2)\, dx$.

89. If f is continuous on $\mathbb{R}$, prove that

$$\int_a^b f(-x)\, dx = \int_{-b}^{-a} f(x)\, dx$$

For the case where $f(x) \geqslant 0$ and $0 < a < b$, draw a diagram to interpret this equation geometrically as an equality of areas.

90. If f is continuous on $\mathbb{R}$, prove that

$$\int_a^b f(x+c)\, dx = \int_{a+c}^{b+c} f(x)\, dx$$

For the case where $f(x) \geqslant 0$, draw a diagram to interpret this equation geometrically as an equality of areas.

91. If a and b are positive numbers, show that

$$\int_0^1 x^a(1-x)^b dx = \int_0^1 x^b(1-x)^a dx$$

92. If f is continuous on $[0, \pi]$, use the substitution $u = \pi - x$ to show that

$$\int_0^\pi xf(\sin x)\, dx = \frac{\pi}{2}\int_0^\pi f(\sin x)\, dx$$

93. Use Exercise 92 to evaluate the integral

$$\int_0^\pi \frac{x \sin x}{1+\cos^2 x}\, dx$$

94.

a. If f is continuous, prove that

$$\int_0^{\pi/2} f(\cos x)\, dx = \int_0^\pi f(\sin x)\, dx$$

b. Use part (a) to evaluate $\int_0^{\pi/2} \cos^2 x\, dx$ and $\int_0^{\pi/2} \sin^2 x\, dx$.

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