Chapter 6: Applications of Integration: 6.1 Exercises
Book Title: Calculus: Early Transcendentals
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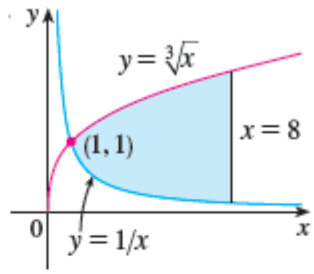
6.1 Exercises

1, 2, 3 and 4 Find the area of the shaded region.

1.



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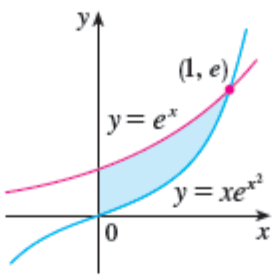

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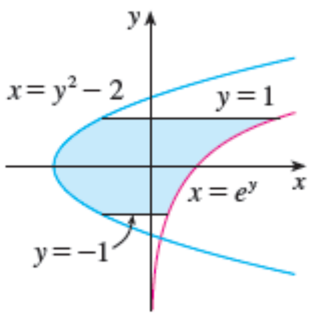

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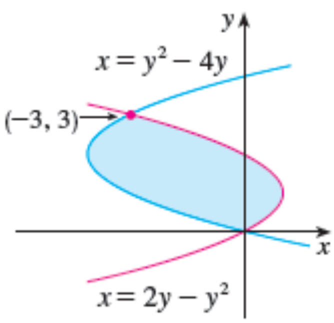


5, 6, 7, 8, 9, 10, 11 and 12 Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

5. $y = e^x, \quad y = x^2 - 1, \quad x = -1, \quad x = 1$

6. $y = \sin x, \quad y = x, \quad x = \pi/2, \quad x = \pi$

7. $y = (x-2)^2, \quad y = x$

8. $y = x^2 - 4x, \quad y = 2x$

9. $y = 1/x, \quad y = 1/x^2, \quad x = 2$

10. $y = \sin x, \quad y = 2x/\pi, \quad x \geqslant 0$

11. $x = 1 - y^2, \quad x = y^2 - 1$

12. $4x + y^2 = 12, \quad x = y$

13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27 and 28 Sketch the region enclosed by the given curves and find its area.

13. $y = 12 - x^2, \quad y = x^2 - 6$

14. $y = x^2, \quad y = 4x - x^2$

15. $y = \sec^2 x, \quad y = 8\cos x, \quad -\pi/3 \leqslant x \leqslant \pi/3$

16. $y = \cos x, \quad y = 2 - \cos x, \quad 0 \leqslant x \leqslant 2\pi$

17. $x = 2y^2, \quad x = 4 + y^2$

18. $y = \sqrt{x-1}, \quad x - y = 1$

19. $y = \cos \pi x, \quad y = 4x^2 - 1$

20. $x = y^4, \quad y = \sqrt{2-x}, \quad y = 0$

21. $y = \tan x, \quad y = 2\sin x, \quad -\pi/3 \leqslant x \leqslant \pi/3$

22. $y = x^3, \quad y = x$

23. $y = \sqrt[3]{2x}, \quad y = \dfrac{1}{8}x^2, \quad 0 \leqslant x \leqslant 6$

24. $y = \cos x, \quad y = 1 - \cos x, \quad 0 \leqslant x \leqslant \pi$

25. $y = x^4, \quad y = 2 - |x|$

26. $y = \sinh x, \quad y = e^{-x}, \quad x = 0, \quad x = 2$

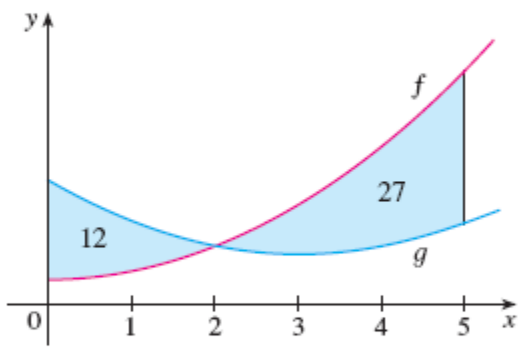
27. $y = 1/x, \quad y = x, \quad y = \dfrac{1}{4}x, \quad x > 0$

28. $y = \dfrac{1}{4}x^2, \quad y = 2x^2, \quad x + y = 3, \quad x \geqslant 0$

29. The graphs of two functions are shown with the areas of the regions between the curves indicated.

a. What is the total area between the curves for $0 \leqslant x \leqslant 5$?

b. What is the value of $\int_0^5 [f(x) - g(x)]\, dx$?



30, 31 and 32 Sketch the region enclosed by the given curves and find its area.

30. $y = \dfrac{x}{\sqrt{1+x^2}}, \quad y = \dfrac{x}{\sqrt{9-x^2}}, \quad x \geqslant 0$

31. $y = \dfrac{x}{1+x^2}, \quad y = \dfrac{x^2}{1+x^3}$

32. $y = \dfrac{\ln x}{x}, \quad y = \dfrac{(\ln x)^2}{x}$

33 and 34 Use calculus to find the area of the triangle with the given vertices.

33. $(0,0), \quad (3,1), \quad (1,2)$

34. $(2, 0), \quad (0, 2), \quad (-1, 1)$

35 and 36 Evaluate the integral and interpret it as the area of a region. Sketch the region.

35. $\displaystyle\int_0^{\pi/2} |\sin x - \cos 2x| \, dx$

36. $\displaystyle\int_{-1}^{1} |3^x - 2^x| \, dx$

37, 38, 39 and 40 Use a graph to find approximate x-coordinates of the points of intersection of the given curves. Then find (approximately) the area of the region bounded by the curves.

37. $y = x \sin\left(x^2\right), \quad y = x^4, \quad x \geqslant 0$

38. $y = \dfrac{x}{\left(x^2+1\right)^2}, \quad y = x^5 - x, \quad x \geqslant 0$

39. $y = 3x^2 - 2x, \quad y = x^3 - 3x + 4$

40. $y = 1.3^x, \quad y = 2\sqrt{x}$

41, 42, 43 and 44 Graph the region between the curves and use your calculator to compute the area correct to five decimal places.

41. $y = \dfrac{2}{1+x^4}, \quad y = x^2$

42. $y = e^{1-x^2}, \quad y = x^4$

43. $y = \tan^2 x, \quad y = \sqrt{x}$

44. $y = \cos x, \quad y = x + 2\sin^4 x$

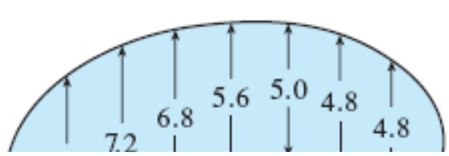
45. CAS Use a computer algebra system to find the exact area enclosed by the curves $y = x^5 - 6x^3 + 4x$ and $y = x$.

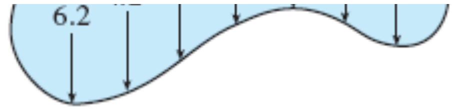
46. Sketch the region in the xy-plane defined by the inequalities $x - 2y^2 \geqslant 0$, $1 - x - |y| \geqslant 0$ and find its area.

47. Racing cars driven by Chris and Kelly are side by side at the start of a race. The table shows the velocities of each car (in miles per hour) during the first ten seconds of the race. Use the Midpoint Rule to estimate how much farther Kelly travels than Chris does during the first ten seconds.

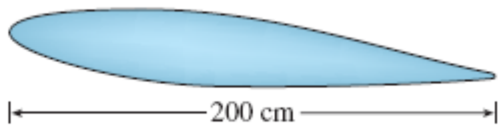
t	v_C	v_K
0	0	0
1	20	22
2	32	37
3	46	52
4	54	61
5	62	71
6	69	80
7	75	86
8	81	93
9	86	98
10	90	102

48. The widths (in meters) of a kidney-shaped swimming pool were measured at 2-meter intervals as indicated in the figure. Use the Midpoint Rule to estimate the area of the pool.





49. A cross-section of an airplane wing is shown. Measurements of the thickness of the wing, in centimeters, at 20-centimeter intervals are 5.8, 20.3, 26.7, 29.0, 27.6, 27.3, 23.8, 20.5, 15.1, 8.7, and 2.8. Use the Midpoint Rule to estimate the area of the wing's cross-section.



50. If the birth rate of a population is $b(t) = 2200e^{0.024t}$ people per year and the death rate is $d(t) = 1460e^{0.018t}$ people per year, find the area between these curves for $0 \leqslant t \leqslant 10$. What does this area represent?

51. In Example 5, we modeled a measles pathogenesis curve by a function f. A patient infected with the measles virus who has some immunity to the virus has a pathogenesis curve that can be modeled by, for instance, $g(t) = 0.9f(t)$.

   a. If the same threshold concentration of the virus is required for infectiousness to begin as in Example 5, on what day does this occur?

   b. Let P_3 be the point on the graph of g where infectiousness begins. It has been shown that infectiousness ends at a point P_4 on the graph of g where the line through P_3, P_4 has the same slope as the line through P_1, P_2 in Example 5(b). On what day does infectiousness end?

   c. Compute the level of infectiousness for this patient.

52. The rates at which rain fell, in inches per hour, in two different locations t hours after the start of a storm are given by $f(t) = 0.73t^3 - 2t^2 + t + 0.6$ and $g(t) = 0.17t^2 - 0.5t + 1.1$. Compute the area between the graphs for $0 \leqslant t \leqslant 2$ and interpret your result in this context.

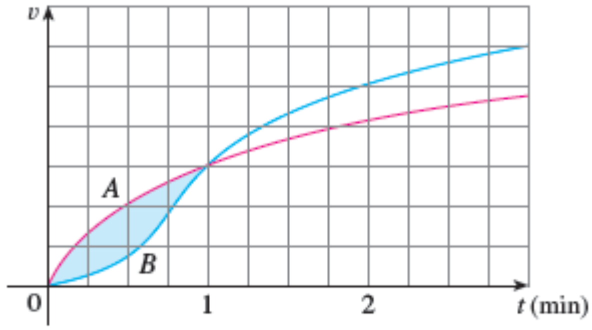
53. Two cars, A and B, start side by side and accelerate from rest. The figure shows the graphs of their velocity functions.

   a. Which car is ahead after one minute? Explain.

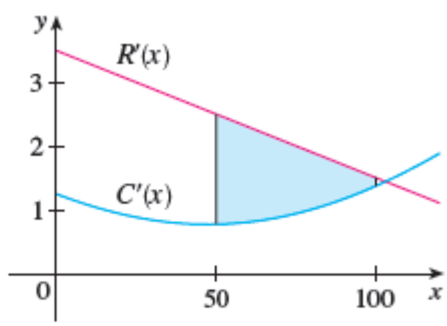
   b. What is the meaning of the area of the shaded region?

   c. Which car is ahead after two minutes? Explain.

d. Estimate the time at which the cars are again side by side.



54. The figure shows graphs of the marginal revenue function R' and the marginal cost function C' for a manufacturer. [Recall from Section 4.7 that $R(x)$ and $C(x)$ represent the revenue and cost when x units are manufactured. Assume that R and C are measured in thousands of dollars.] What is the meaning of the area of the shaded region? Use the Midpoint Rule to estimate the value of this quantity.



55. The curve with equation $y^2 = x^2(x+3)$ is called **Tschirnhausen's cubic**. If you graph this curve you will see that part of the curve forms a loop. Find the area enclosed by the loop.

56. Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1,1)$, and the x-axis.

57. Find the number b such that the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 4$ into two regions with equal area.

58.

a. Find the number a such that the line $x = a$ bisects the area under the curve $y = 1/x^2$, $1 \leqslant x \leqslant 4$.

b. Find the number b such that the line $y = b$ bisects the area in part (a).

59. Find the values of c such that the area of the region bounded by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is 576.

60. Suppose that $0 < c < \pi/2$. For what value of c is the area of the region enclosed by the curves $y = \cos x$, $y = \cos(x - c)$, and $x = 0$ equal to the area of the region enclosed by the curves $y = \cos(x - c)$, $x = \pi$, and $y = 0$?

61. For what values of m do the line $y = mx$ and the curve $y = x/(x^2 + 1)$ enclose a region? Find the area of the region.

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