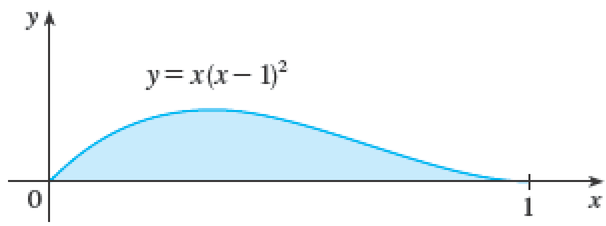
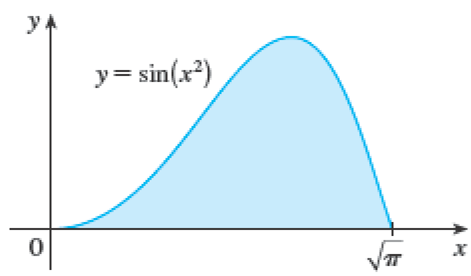
Chapter 6: Applications of Integration: 6.3 Exercises
Book Title: Calculus: Early Transcendentals
Printed By: Jordan Hoffart (jordanhoffart@tamu.edu)

6.3 Exercises

1. Let S be the solid obtained by rotating the region shown in the figure about the y-axis. Explain why it is awkward to use slicing to find the volume V of S. Sketch a typical approximating shell. What are its circumference and height? Use shells to find V.



2. Let S be the solid obtained by rotating the region shown in the figure about the y-axis. Sketch a typical cylindrical shell and find its circumference and height. Use shells to find the volume of S. Do you think this method is preferable to slicing? Explain.



3, 4, 5, 6 and 7 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis.

3. $y = \sqrt[3]{x}, \quad y = 0, \quad x = 1$

4. $y = x^3, \quad y = 0, \quad x = 1, \quad x = 2$

5. $y = e^{-x^2}, \quad y = 0, \quad x = 0, \quad x = 1$

6. $y = 4x - x^2, \quad y = x$

7. $y = x^2, \quad y = 6x - 2x^2$

8. Let V be the volume of the solid obtained by rotating about the y-axis the region bounded by $y = \sqrt{x}$ and $y = x^2$. Find V both by slicing and by cylindrical shells. In both cases draw a diagram to explain your method.

9, 10, 11, 12, 13 and 14 Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis.

9. $xy = 1, \quad x = 0, \quad y = 1, \quad y = 3$

10. $y = \sqrt{x}, \quad x = 0, \quad y = 2$

11. $y = x^{3/2}, \quad y = 8, \quad x = 0$

12. $x = -3y^2 + 12y - 9, \quad x = 0$

13. $x = 1 + (y-2)^2, \quad x = 2$

14. $x + y = 4, \quad x = y^2 - 4y + 4$

15, 16, 17, 18, 19 and 20 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis.

15. $y = x^3$, $y = 8$, $x = 0$; about $x = 3$

16. $y = 4 - 2x$, $y = 0$, $x = 0$; about $x = -1$

17. $y = 4x - x^2$, $y = 3$; about $x = 1$

18. $y = \sqrt{x}$, $x = 2y$; about $x = 5$

19. $x = 2y^2$, $y \geqslant 0$, $x = 2$; about $y = 2$

20. $x = 2y^2$, $x = y^2 + 1$; about $y = -2$

21, 22, 23, 24, 25 and 26

a. Set up an integral for the volume of the solid obtained by rotating the region bounded by the given curve about the specified axis.

b. Use your calculator to evaluate the integral correct to five decimal places.

21. $y = xe^{-x}$, $y = 0$, $x = 2$; about the y-axis

22. $y = \tan x$, $y = 0$, $x = \pi/4$; about $x = \pi/2$

23. $y = \cos^4 x$, $y = -\cos^4 x$, $-\pi/2 \leqslant x \leqslant \pi/2$; about $x = \pi$

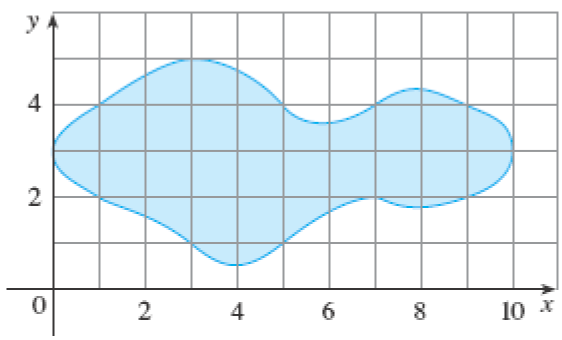
24. $y = x$, $y = 2x/\left(1 + x^3\right)$; about $x = -1$

25. $x = \sqrt{\sin y}$, $0 \leqslant y \leqslant \pi$, $x = 0$; about $y = 4$

26. $x^2 - y^2 = 7$, $x = 4$; about $y = 5$

27. Use the Midpoint Rule with $n = 5$ to estimate the volume obtained by rotating about the y-axis the region under the curve $y = \sqrt{1 + x^3}$, $0 \leqslant x \leqslant 1$.

28. If the region shown in the figure is rotated about the y-axis to form a solid, use the Midpoint Rule with $n = 5$ to estimate the volume of the solid.



29, 30, 31 and 32 Each integral represents the volume of a solid. Describe the solid.

29. $\displaystyle\int_0^3 2\pi x^5 \, dx$

30. $\displaystyle\int_1^3 2\pi \, y \ln y \, dy$

31. $\displaystyle 2\pi \int_1^4 \frac{y + 2}{y^2} dy$

32. $\displaystyle\int_0^1 2\pi \left(2 - x\right) \left(3^x - 2^x\right) \, dx$

33 and 34 Use a graph to estimate the x-coordinates of the points of intersection of the given curves. Then use this information and your calculator to estimate the volume of the solid obtained by rotating about the y-axis the region enclosed by these curves.

33. $y = x^2 - 2x, \quad y = \dfrac{x}{x^2+1}$

34. $y = e^{\sin x}, \quad y = x^2 - 4x + 5$

35 and 36 CAS Use a computer algebra system to find the exact volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

35. $y = \sin^2 x$, $y = \sin^4 x$, $0 \leqslant x \leqslant \pi$; about $x = \pi/2$

36. $y = x^3 \sin x$, $y = 0$, $0 \leqslant x \leqslant \pi$; about $x = -1$

37, 38, 39, 40, 41, 42 and 43 The region bounded by the given curves is rotated about the specified axis. Find the volume of the resulting solid by any method.

37. $y = -x^2 + 6x - 8$, $y = 0$; about the y-axis

38. $y = -x^2 + 6x - 8$, $y = 0$; about the x-axis

39. $y^2 - x^2 = 1$, $y = 2$; about the x-axis

40. $y^2 - x^2 = 1$, $y = 2$; about the y-axis

41. $x^2 + (y-1)^2 = 1$; about the y-axis

42. $x = (y-3)^2$, $x = 4$; about $y = 1$

43. $x = (y-1)^2$, $x - y = 1$; about $x = -1$

44. Let T be the triangular region with vertices $(0{,}0)$, $(1{,}0)$, and $(1, 2)$, and let V be the volume of the solid generated when T is rotated about the line $x = a$, where $a > 1$. Express a in terms of V.

45, 46 and 47 Use cylindrical shells to find the volume of the solid.

45. A sphere of radius r

46. The solid torus of Exercise 6.2.63

47. A right circular cone with height h and base radius r

48. Suppose you make napkin rings by drilling holes with different diameters through two wooden balls (which also have different diameters). You discover that both napkin rings have the same height h, as shown in the figure.

    a. Guess which ring has more wood in it.

    b. Check your guess: Use cylindrical shells to compute the volume of a napkin ring created by drilling a hole with radius r through the center of a sphere of radius R and express the answer in terms of h.

Chapter 6: Applications of Integration: 6.3 Exercises
Book Title: Calculus: Early Transcendentals
Printed By: Jordan Hoffart (jordanhoffart@tamu.edu)