Chapter 11: Infinite Sequences and Series: 11.4 Exercises
Book Title: Calculus: Early Transcendentals
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# 11.4 Exercises

1. Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is known to be convergent.

   a. If $a_n > b_n$ for all $n$, what can you say about $\sum a_n$? Why?

   b. If $a_n < b_n$ for all $n$, what can you say about $\sum a_n$? Why?

2. Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is known to be divergent.

   a. If $a_n > b_n$ for all $n$, what can you say about $\sum a_n$? Why?

   b. If $a_n < b_n$ for all $n$, what can you say about $\sum a_n$? Why?

3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31 and 32 Determine whether the series converges or diverges.

3. $\sum_{n=1}^{\infty} \frac{1}{n^3 + 8}$

4. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n} - 1}$

5. $\sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}}$

6. $\sum_{n=1}^{\infty} \frac{n-1}{n^3 + 1}$

7. $\sum_{n=1}^{\infty} \frac{9^n}{3 + 10^n}$

8. $\sum_{n=1}^{\infty} \frac{6^n}{5^n - 1}$

9. $\sum_{k=1}^{\infty} \frac{\ln k}{k}$

10. $\sum_{k=1}^{\infty} \frac{k \sin^2 k}{1+k^3}$

11. $\sum_{k=1}^{\infty} \frac{\sqrt[3]{k}}{\sqrt{k^3+4k+3}}$

12. $\sum_{k=1}^{\infty} \frac{(2k-1)\left(k^2-1\right)}{(k+1)\left(k^2+4\right)^2}$

13. $\sum_{n=1}^{\infty} \frac{1+\cos n}{e^n}$

14. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{3n^4+1}}$

15. $\sum_{n=1}^{\infty} \frac{4^{n+1}}{3^n-2}$

16. $\sum_{n=1}^{\infty} \frac{1}{n^n}$

17. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$

18. $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}+2}$

19. $\sum_{n=1}^{\infty} \frac{n+1}{n^3+n}$

20. $\sum_{n=1}^{\infty} \frac{n^2+n+1}{n^4+n^2}$

21. $\sum_{n=1}^{\infty} \frac{\sqrt{1+n}}{2+n}$

22. $\sum_{n=3}^{\infty} \frac{n+2}{(n+1)^3}$

23. $\sum_{n=1}^{\infty} \frac{5+2n}{\left(1+n^2\right)^2}$

24. $\sum_{n=1}^{\infty} \frac{n+3^n}{n+2^n}$

25.

$$\sum_{n=1}^{\infty} \frac{e^n + 1}{ne^n + 1}$$

26. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2 - 1}}$

27. $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^2 e^{-n}$

28. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$

29. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$

30. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

31. $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

32. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$

33, 34, 35 and 36 Use the sum of the first $10$ terms to approximate the sum of the series. Estimate the error.

33. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5 + n^5}$

34. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^4}$

35. $\displaystyle\sum_{n=1}^{\infty} 5^{-n} \cos^2 n$

36. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n + 4^n}$

37. The meaning of the decimal representation of a number $0.d_1 d_2 d_3 \ldots$ (where the digit $d_i$ is one of the numbers $0,\ 1, 2, \ldots,\ 9$) is that

$$0.d_1d_2d_3d_4\ldots = \frac{d_1}{10} + \frac{d_2}{10^2} + \frac{d_3}{10^3} + \frac{d_4}{10^4} + \cdots$$

Show that this series always converges.

38. For what values of $p$ does the series $\sum_{n=2}^{\infty} 1/(n^p \ln n)$ converge?

39. Prove that if $a_n \geqslant 0$ and $\sum a_n$ converges, then $\sum a_n^2$ also converges.

40.

a. Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is convergent. Prove that if

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 0$$

then $\sum a_n$ is also convergent.

b. Use part (a) to show that the series converges.

i. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

ii. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n}\, e^n}$

41.

a. Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is divergent. Prove that if

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$$

then $\sum a_n$ is also divergent.

b. Use part (a) to show that the series diverges.

i. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$

ii. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

42. Give an example of a pair of series $\sum a_n$ and $\sum b_n$ with positive terms where $\lim_{n\to\infty} (a_n/b_n) = 0$ and $\sum b_n$ diverges, but $\sum a_n$ converges. (Compare with Exercise 40.)

43. Show that if $a_n > 0$ and $\lim_{n\to\infty} na_n \neq 0$, then $\sum a_n$ is divergent.

44. Show that if $a_n > 0$ and $\sum a_n$ is convergent, then $\sum \ln(1 + a_n)$ is convergent.

45. If $\sum a_n$ is a convergent series with positive terms, is it true that $\sum \sin(a_n)$ is also convergent?

46. If $\sum a_n$ and $\sum b_n$ are both convergent series with positive terms, is it true that $\sum a_n b_n$ is also convergent?

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