Chapter 7: Techniques of Integration: 7.4 Exercises
Book Title: Calculus: Early Transcendentals
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7.4 Exercises

1, 2, 3, 4, 5 and 6 Write out the form of the partial fraction decomposition of the function (as in Example 7). Do not determine the numerical values of the coefficients.

1.
   a. $\dfrac{4+x}{(1+2x)(3-x)}$
   b. $\dfrac{1-x}{x^3+x^4}$

2.
   a. $\dfrac{x-6}{x^2+x-6}$
   b. $\dfrac{x^2}{x^2+x+6}$

3.
   a. $\dfrac{1}{x^2+x^4}$
   b. $\dfrac{x^3+1}{x^3-3x^2+2x}$

4.
   a. $\dfrac{x^4-2x^3+x^2+2x-1}{x^2-2x+1}$
   b. $\dfrac{x^2-1}{x^3+x^2+x}$

5.
   a. $\dfrac{x^6}{x^2-4}$
   b. $\dfrac{x^4}{(x^2-x+1)(x^2+2)^2}$

6.
   a. $\dfrac{t^6+1}{t^6+t^3}$

b. $\dfrac{x^5+1}{(x^2-x)(x^4+2x^2+1)}$

7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37 and 38 Evaluate the integral.

7. $\displaystyle\int \frac{x^4}{x-1}dx$

8. $\displaystyle\int \frac{3t-2}{t+1}dt$

9. $\displaystyle\int \frac{5x+1}{(2x+1)(x-1)}dx$

10. $\displaystyle\int \frac{y}{(y+4)(2y-1)}dy$

11. $\displaystyle\int_0^1 \frac{2}{2x^2+3x+1}dx$

12. $\displaystyle\int_0^1 \frac{x-4}{x^2-5x+6}dx$

13. $\displaystyle\int \frac{ax}{x^2-bx}dx$

14. $\displaystyle\int \frac{1}{(x+a)(x+b)}dx$

15. $\displaystyle\int_{-1}^0 \frac{x^3-4x+1}{x^2-3x+2}dx$

16. $\displaystyle\int_1^2 \frac{x^3+4x^2+x-1}{x^3+x^2}dx$

17. $\displaystyle\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}dy$

18. $\displaystyle\int_1^2 \frac{3x^2+6x+2}{x^2+3x+2}dx$

19. $\displaystyle\int_0^1 \frac{x^2+x+1}{(x+1)^2(x+2)}dx$

20. $\displaystyle\int_2^3 \frac{x(3-5x)}{(3x-1)(x-1)^2}dx$

21. $\displaystyle\int \frac{dt}{(t^2-1)^2}$

22. $\int \frac{x^4 + 9x^2 + x + 2}{x^2 + 9} dx$

23. $\int \frac{10}{(x-1)(x^2+9)} dx$

24. $\int \frac{x^2 - x + 6}{x^3 + 3x} dx$

25. $\int \frac{4x}{x^3 + x^2 + x + 1} dx$

26. $\int \frac{x^2 + x + 1}{(x^2+1)^2} dx$

27. $\int \frac{x^3 + 4x + 3}{x^4 + 5x^2 + 4} dx$

28. $\int \frac{x^3 + 6x - 2}{x^4 + 6x^2} dx$

29. $\int \frac{x + 4}{x^2 + 2x + 5} dx$

30. $\int \frac{x^3 - 2x^2 + 2x - 5}{x^4 + 4x^2 + 3} dx$

31. $\int \frac{1}{x^3 - 1} dx$

32. $\int_0^1 \frac{x}{x^2 + 4x + 13} dx$

33. $\int_0^1 \frac{x^3 + 2x}{x^4 + 4x^2 + 3} dx$

34. $\int \frac{x^5 + x - 1}{x^3 + 1} dx$

35. $\int \frac{5x^4 + 7x^2 + x + 2}{x(x^2+1)^2} dx$

36. $\int \frac{x^4 + 3x^2 + 1}{x^5 + 5x^3 + 5x} dx$

37. $\int \frac{x^2 - 3x + 7}{(x^2 - 4x + 6)^2} dx$

38. $\int \frac{x^3 + 2x^2 + 3x - 2}{(x^2 + 2x + 2)^2} dx$

39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 and 52 Make a substitution to express the integrand as a rational function and then evaluate the integral.

39. $\int \frac{dx}{x\sqrt{x-1}}$

40. $\int \frac{dx}{2\sqrt{x+3}+x}$

41. $\int \frac{dx}{x^2+x\sqrt{x}}$

42. $\int_0^1 \frac{1}{1+\sqrt[3]{x}}dx$

43. $\int \frac{x^3}{\sqrt[3]{x^2+1}}dx$

44. $\int \frac{dx}{(1+\sqrt{x})^2}$

45. $\int \frac{1}{\sqrt{x}-\sqrt[3]{x}}dx$ [*Hint:* Substitute $u=\sqrt[6]{x}$.]

46. $\int \frac{\sqrt{1+\sqrt{x}}}{x}dx$

47. $\int \frac{e^{2x}}{e^{2x}+3e^x+2}dx$

48. $\int \frac{\sin x}{\cos^2 x-3\cos x}dx$

49. $\int \frac{\sec^2 t}{\tan^2 t+3\tan t+2}dt$

50. $\int \frac{e^x}{(e^x-2)(e^{2x}+1)}dx$

51. $\int \frac{dx}{1+e^x}$

52. $\int \frac{\cosh t}{\sinh^2 t+\sinh^4 t}dt$

53 and 54 Use integration by parts, together with the techniques of this section, to evaluate the integral.

53. $\int \ln\left(x^2 - x + 2\right)\,dx$

54. $\int x\tan^{-1}x\,dx$

55. Use a graph of $f(x) = 1/\left(x^2 - 2x - 3\right)$ to decide whether $\int_0^2 f(x)\,dx$ is positive or negative. Use the graph to give a rough estimate of the value of the integral and then use partial fractions to find the exact value.

56. Evaluate

$$\int \frac{1}{x^2 + k}dx$$

by considering several cases for the constant k.

57 and 58 Evaluate the integral by completing the square and using Formula 6.

57. $\int \frac{dx}{x^2 - 2x}$

58. $\int \frac{2x + 1}{4x^2 + 12x - 7}dx$

59. The German mathematician Karl Weierstrass (1815–1897) noticed that the substitution $t = \tan(x/2)$ will convert any rational function of $\sin x$ and $\cos x$ into an ordinary rational function of t.

a. If $t = \tan(x/2)$, $-\pi < x < \pi$, sketch a right triangle or use trigonometric identities to show that

$$\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1 + t^2}}$$

and

$$\sin\left(\frac{x}{2}\right) = \frac{t}{\sqrt{1 + t^2}}$$

b. Show that

$$\cos x = \frac{1 - t^2}{1 + t^2}$$

and

$$\sin x = \frac{2t}{1+t^2}$$

c. Show that

$$dx = \frac{2}{1+t^2}dt$$

60, 61, 62 and 63 Use the substitution in Exercise 59 to transform the integrand into a rational function of t and then evaluate the integral.

60. $\int \frac{dx}{1-\cos x}$

61. $\int \frac{1}{3\sin x - 4\cos x}dx$

62. $\int_{\pi/3}^{\pi/2} \frac{1}{1+\sin x - \cos x}dx$

63. $\int_{0}^{\pi/2} \frac{\sin 2x}{2+\cos x}dx$

64 and 65 Find the area of the region under the given curve from 1 to 2.

64. $y = \frac{1}{x^3+x}$

65. $y = \frac{x^2+1}{3x-x^2}$

66. Find the volume of the resulting solid if the region under the curve $y = 1/(x^2+3x+2)$ from $x=0$ to $x=1$ is rotated about

a. the x-axis and

b. the y-axis.

67. One method of slowing the growth of an insect population without using pesticides is to introduce into the population a number of sterile males that mate with fertile females but produce no offspring. (The photo shows a screw-worm fly, the first pest effectively eliminated from a region by this method.)

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Let P represent the number of female insects in a population and S the number of sterile males introduced each generation. Let r be the per capita rate of production of females by females, provided their chosen mate is not sterile. Then the female population is related to time t by

$$t = \int \frac{P+S}{P\left[(r-1)P-S\right]}\, dP$$

Suppose an insect population with $10{,}000$ females grows at a rate of $r = 1.1$ and 900 sterile males are added initially. Evaluate the integral to give an equation relating the female population to time. (Note that the resulting equation can't be solved explicitly for P.)

68. Factor $x^4 + 1$ as a difference of squares by first adding and subtracting the same quantity. Use this factorization to evaluate $\int 1/(x^4+1)\, dx$.

69.

a. Use a computer algebra system to find the partial fraction decomposition of the function

$$f(x) = \frac{4x^3 - 27x^2 + 5x - 32}{30x^5 - 13x^4 + 50x^3 - 286x^2 - 299x - 70}$$

b. Use part (a) to find $\int f(x)\, dx$ (by hand) and compare with the result of using the CAS to integrate f directly. Comment on any discrepancy.

70. CAS

a. Find the partial fraction decomposition of the function

$$f(x) = \frac{12x^5 - 7x^3 - 13x^2 + 8}{100x^6 - 80x^5 + 116x^4 - 80x^3 + 41x^2 - 20x + 4}$$

b. Use part (a) to find $\int f(x)\, dx$ and graph f and its indefinite integral on the same screen.

c. Use the graph of f to discover the main features of the graph of

$$\int f(x)\,dx.$$

71. The rational number $\frac{22}{7}$ has been used as an approximation to the number π since the time of Archimedes. Show that

$$\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx = \frac{22}{7} - \pi$$

72.

a. Use integration by parts to show that, for any positive integer n,

$$\int \frac{dx}{(x^2+a^2)^n}dx = \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} + \frac{2n-3}{2a^2(n-1)}\int \frac{dx}{(x^2+a^2)^{n-1}}$$

b. Use part (a) to evaluate

$$\int \frac{dx}{(x^2+1)^2}$$

and

$$\int \frac{dx}{(x^2+1)^3}$$

73. Suppose that F, G, and Q are polynomials and

$$\frac{F(x)}{Q(x)} = \frac{G(x)}{Q(x)}$$

for all x except when $Q(x) = 0$. Prove that $F(x) = G(x)$ for all x. [*Hint:* Use continuity.]

74. If f is a quadratic function such that $f(0) = 1$ and

$$\int \frac{f(x)}{x^2(x+1)^3}dx$$

is a rational function, find the value of $f'(0)$.

75. If $a \neq 0$ and n is a positive integer, find the partial fraction decomposition of

$$f(x) = \frac{1}{x^n(x-a)}$$

[*Hint:* First find the coefficient of $1/(x-a)$. Then subtract the resulting term and simplify what is left.]

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