14. $\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx$

Let $I(t) = \int_1^t \frac{1}{x^2} e^{-1/x}\,dx$

$t > 1$

$$= \int_{\cancel{-1/t}\ -1}^{\cancel{-1}\ -1/t} e^u\,du = \cancel{\left\{e^{-1} - e^{-1/t}\right\}}$$

$$e^{-1/t} - e^{-1}$$

$u = -\frac{1}{x}$

$du = \frac{1}{x^2}\,dx$

~~Then $I_1(t) \to e^{-1}$ as $t \to \infty$, so~~

~~$\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx = e^{-1}$~~

$I_1(t) \to 1 - e^{-1}$ as $t \to \infty$, so

$$\int_1^\infty \frac{e^{-\frac{1}{x}}}{x^2}\,dx = 1 - \frac{1}{e}$$

$\square$

31. $\int_{-2}^{3} \frac{1}{x^4}\,dx$

$I_1(t) = \int_{-2}^t \frac{1}{x^4}\,dx$

$-2 < t < 0$

$$= -\frac{1}{3x^3}\Big|_{-2}^{t} = -\frac{1}{3}\left\{\frac{1}{t^3} + \frac{1}{2}\right\} \to +\infty$$

as $t \to 0^-$

Thus the integral DNE.

50. $\displaystyle\int_1^\infty \frac{1+\sin^2 x}{\sqrt{x}}\,dx$ ← Does this converge / diverge?

Use the comparison test.

Observe that $0 \le \sin^2 x \le 1$ for all x, so

$$0 \le \frac{1}{\sqrt{x}} \le \frac{1+\sin^2 x}{\sqrt{x}} \le \frac{2}{\sqrt{x}} \quad \text{for all } x \ge 1.$$

Now $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}}\,dx = 2\lim_{t\to\infty}\left(t^{1/2} - 1\right) = \infty$,

so the integral <u>diverges</u> to ∞.