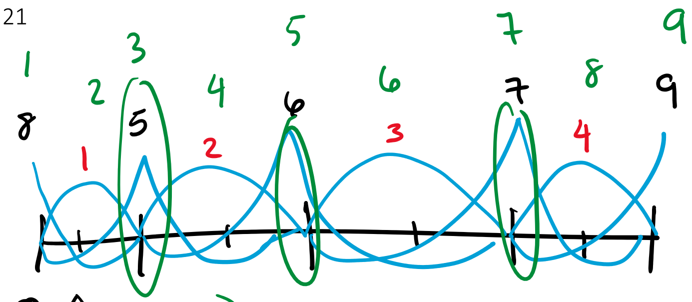
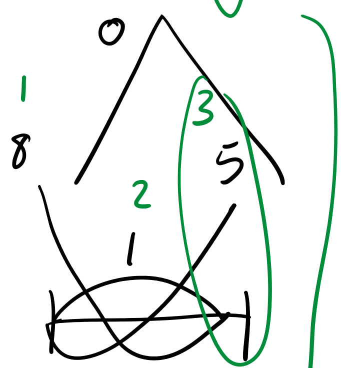
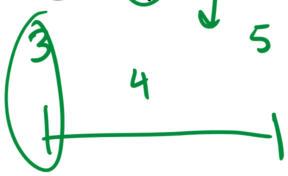
610 Recitation 2024 02 21

Wednesday, February 21, 2024 7:54 AM



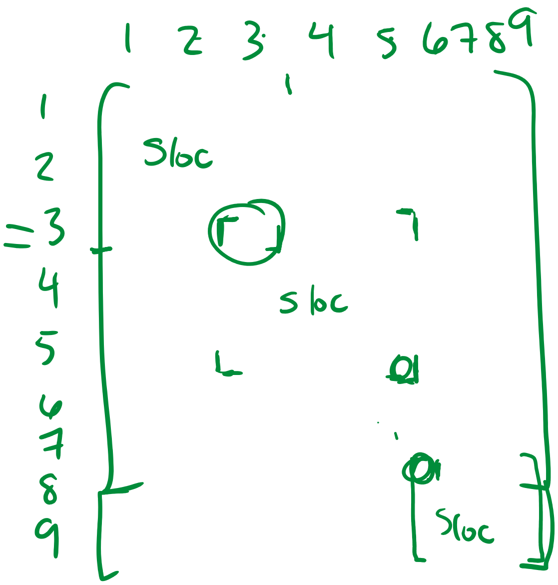




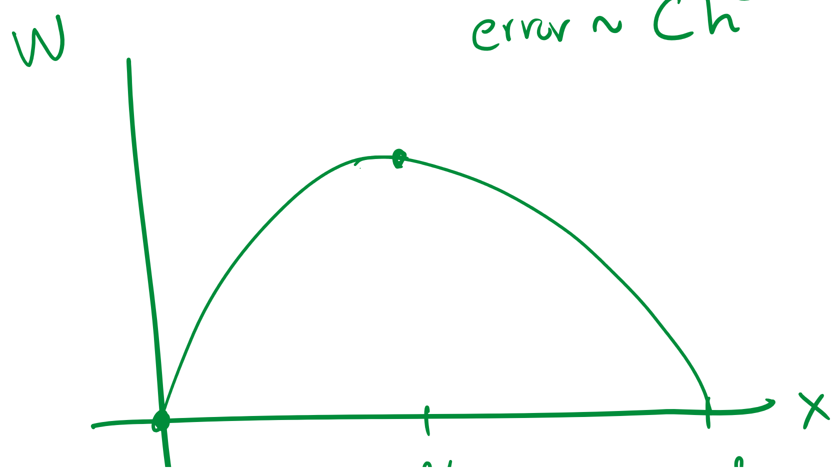
$$S = \begin{array}{cc} & \begin{array}{ccc}\varphi_1 & \cdots & \varphi_9\end{array} \\ \begin{array}{c}\varphi_1\\ \vdots \\ \varphi_9\end{array} & \begin{bmatrix} & & \\ & \int \varphi_j' \varphi_i' & \\ & & \end{bmatrix}\end{array}$$

$$S_{loc} = \begin{array}{cc} \begin{array}{ccc}\hat{\varphi}_1 & \hat{\varphi}_2 & \hat{\varphi}_3\end{array} & \\ \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} & \begin{array}{c}\hat{\varphi}_1\\ \hat{\varphi}_2\\ \hat{\varphi}_3\end{array}\end{array}$$

$S =$



error $\sim C h^{r}$

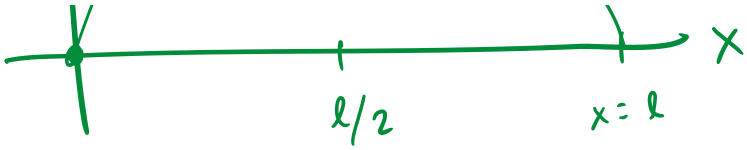


Error rates

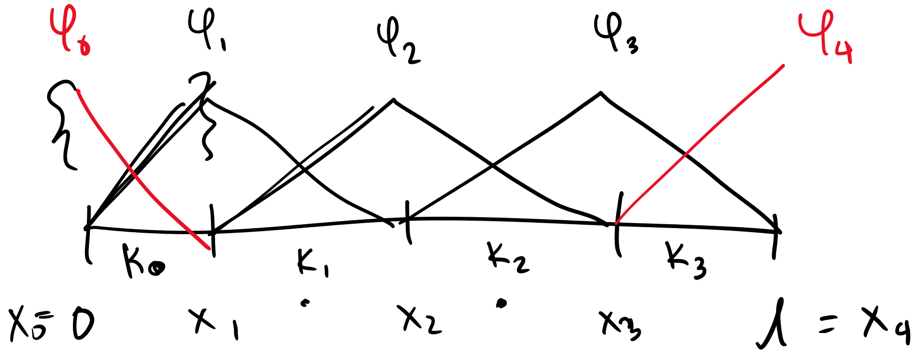
$L^2 \sim 2$

$H^1 \sim 1$

$L^\infty \sim 1$

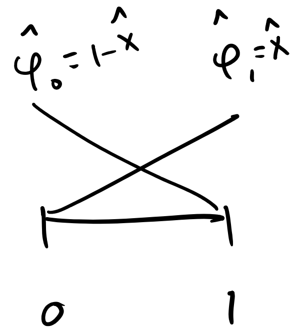


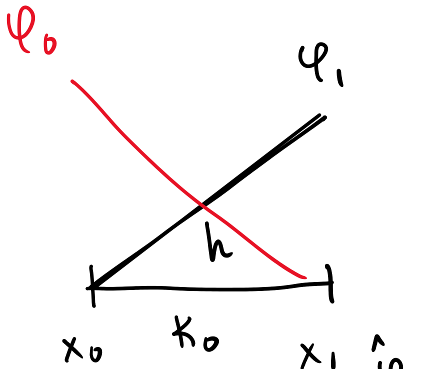
$L^\infty \sim 1$



$N = 4$

$$A_h = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & & & & \\ 1 & & \bullet & & \\ 2 & & & a(\varphi_j, \varphi_i) & \\ 3 & & & & \end{array}$$

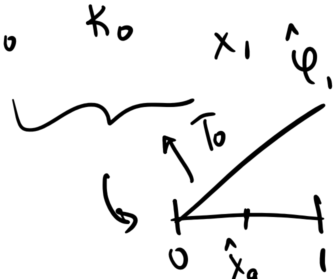




$$a(\varphi_1, \varphi_1) = \int_0^l \varphi_1^2 \, dx$$

$x = T_0 \hat x$

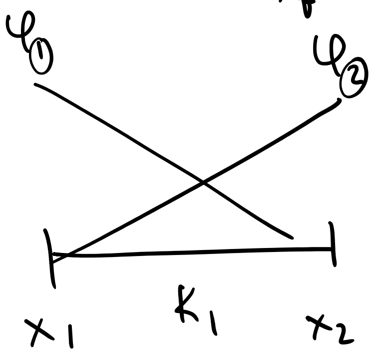
$$= \int_{K_0} \varphi_1^2 + \int_{K_1} \varphi_1^2 \quad \text{" ... "}$$



$\varphi_1 \sim \hat\varphi_1$

local contributions from K_0

to $a(\varphi_1, \varphi_1)$ from K_1



$a(\varphi_1, \varphi_1$

$\varphi_1 \sim \hat\varphi_0$

local contributions from K_1 :

$$\begin{matrix} a(\varphi_1, \varphi_1) & a(\varphi_2, \varphi_1) \\ a(\varphi_1, \varphi_2) & a(\varphi_2, \varphi_2) \end{matrix}$$

Pull back to reference element, the integrals reduce to computing $\underline{\underline{\int_0^1 \hat{\varphi}_0^2}}$, $\underline{\underline{\int_0^1 \hat{\varphi}_0 \hat{\varphi}_1}}$, and $\underline{\underline{\int_0^1 \hat{\varphi}_1^2}}$

$$\hat{A} = \begin{bmatrix} \int_0^1 \hat{\varphi}_0^2 & \int_0^1 \hat{\varphi}_0 \hat{\varphi}_1 \\ \int_0^1 \hat{\varphi}_0 \hat{\varphi}_1 & \int_0^1 \hat{\varphi}_1^2 \end{bmatrix} \qquad A_h = h\hat{A}$$

Then

$$A = \begin{bmatrix} [A_h] & & \\ & [A_h] & \\ & & [A_h] \end{bmatrix}$$

Assemble without applying BC's

$$A \quad \begin{array}{c} \\ \varphi_0 \\ \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{array} \begin{array}{c} \begin{array}{ccccc} \varphi_0 & \varphi_1 & \varphi_2 & \varphi_3 & \varphi_4 \end{array} \\ \left[\begin{array}{c|ccc|c} 1 & 0 & \text{---} & & 0 \\ \hline & & & & \\ & & & & \\ & & & & \\ \hline 0 & \text{---} & & & 1 \end{array}\right] \end{array} \qquad F = \left[\begin{array}{c} 0 \\ \hline \\ \\ \\ \hline 0 \end{array}\right] \begin{array}{c} \varphi_0 \\ \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{array}$$

Then either 1. delete row/col that's not needed

or

2. set diagonal enty to 1, rest of row to 0, RHS enty to 0 like above