610 Recitation 2024 02 28

Wednesday, February 28, 2024 7:52 AM

$$e_h \sim C h^r$$

$$e_1 \sim C h_1^r$$

$$e_2 \sim C h_2^r$$

$$\frac{e_1}{e_2} \sim \left(\frac{h_1}{h_2}\right)^r$$

$$\log(e_1/e_2) \sim r \log(h_1/h_2)$$

$$\boxed{\frac{\log(e_1/e_2)}{\log(h_1/h_2)} \sim r}$$

$$\|f\|_{L^2}^2 = \int f(x)^2 dx \sim C h^2$$

$$\|f'\|_{L^2}^2 = \int f'(x)^2 dx \qquad f'(x) \sim \frac{f(x+h) - f(x)}{h}$$

$$\sim C h^{2(r-1)}$$

$$\|f\|_{H^1} = \left(\|f\|_{L^2}^2 + \|f'\|_{L^2}^2\right)^{1/2}$$

P4 in CA 1

$$-(ku')' = 0$$

$$V \overset{!}{=} \cancel{\{u \in H^1 \mid u(0) = 0\}}$$

$$-(ku')' = 0$$

$$u(0) = 0$$

$$u(1) = \alpha \neq 0$$

~~$V := \{ u \in H^1 \mid u(0) = 0,\ u(1) = \alpha \}$~~

Not a vector space!

Goal: Find w s.t. $w(1) = \alpha$, $w(0) = 0$

eg $\boxed{w(x) = \alpha x}$ *

Now write $u(x) = w(x) + \underbrace{v(x)}_{:= u(x) - w(x)}$ *

and insert into the ODE

$$-(ku')' = -(k(w' + v'))' = -(k(\alpha + v'))'$$

$$= -\alpha k' - (kv')' = 0$$

$$u(0) = w(0) + v(0) = 0 \rightarrow v(0) = 0$$

$$u(1) = \underbrace{w(1)}_{=\alpha} + v(1) = \alpha \rightarrow v(1) = 0$$

Conclusion: v satisfies

$$\begin{cases} -(kv')' = \underline{\underline{\alpha k'}} \end{cases}$$

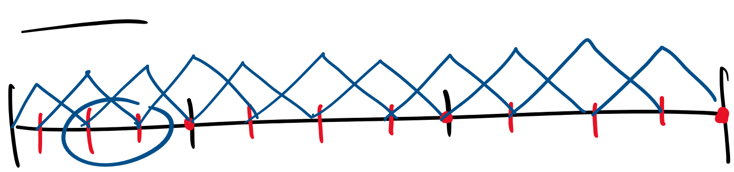
$$\ast \begin{cases} -(kv')' = \dots \\ v(0) = 0 \\ v(1) = 0 \end{cases} \longleftarrow \underline{\text{Homogeneous!}}$$

Step 1: Lift BC ✓

Step 2: Approximate v w/ this problem to get v_h

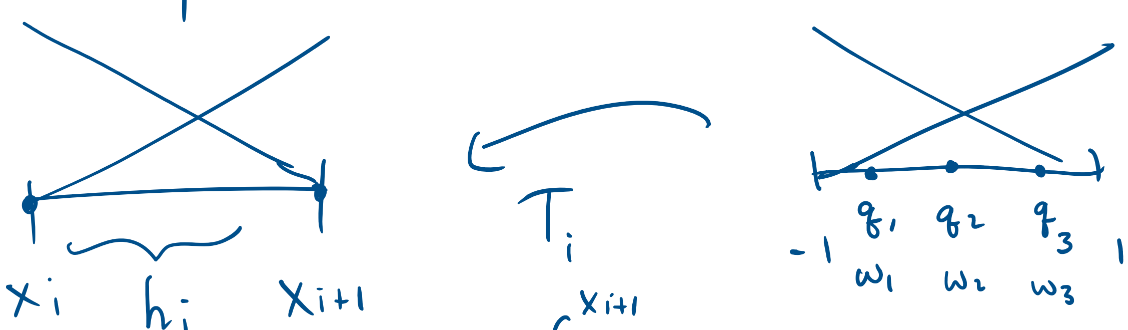
Step 3: Set $u_h = w + v_h$ to ~~approximate~~ u.

$$k(x) = \begin{cases} 1 \\ 2 \\ 3 \end{cases}$$



$$-(ku')' + qu = f$$

$k \in L^\infty$ and $k \geq k_0 > 0$



$$\int^{x_{i+1}} f(x)\,dx \approx \sum f(T_i(q_i))\,w_i\,T_i' \dots$$

w_3

$x_i \qquad h_i \qquad x_{i+1}$

$$\int_{x_i}^{x_{i+1}} f(x)\,dx \sim \sum_j f(T_i(q_j))\,w_j\,T_i'(q_j)$$