610 HW 4

1. Max principle

If $\left(-\frac{W_{i+1}-2W_i+W_{i-1}}{h^2}+b\frac{W_{i+1}-W_i}{h}\right)\leq 0$

$$W_0 = W_{N+1} = 0$$

<u>and</u> $h \leq \bar{h}$,

then

$$\max_i W_i = \max(W_0, W_{N+1}) = 0$$

2. Minimum principle

If $(\text{Discretization}) \geq 0$

$$W_0 = W_{N+1} = 0$$

and $h \leq \bar{h}$

then $\min_i W_i = \min(W_0, W_{N+1}) = 0$

3. The system

$$-\frac{W_{i+1}-2W_i+W_{i-1}}{h^2}+6\frac{W_{i+1}-W_i}{h}=0$$

$$W_{N+1}=W_0=0$$

only has $W_i=0$ as its solution.

4. Taylor's theorem Under certain smoothness assumptions of W

$$W(x+h)=W(x)+hW'(x)+\frac{h^2}{2}W''(x)+\sum_k \frac{h^k}{k!}W^{(k)}(x)$$

$$W(x-h)=W(x)-hW'(x)+\frac{h^2}{2}W''(x)+\sum_k \frac{(-1)^k h^k}{k!}W^{(k)}(x)$$

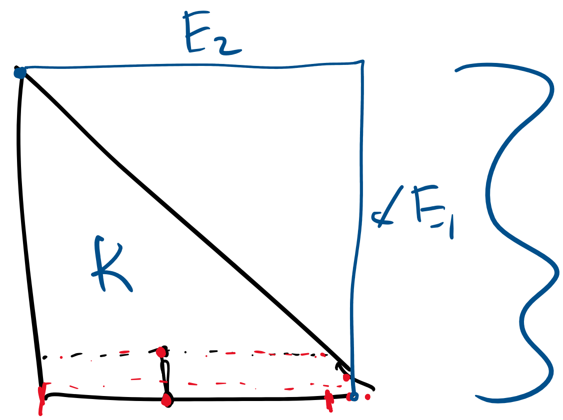
$$\rightarrow \quad W(x+h)+W(x-h)=2W(x)+h^2W''(x)+O(h^4)$$

$$\rightarrow \boxed{\frac{W(x+h) - 2W(x) + W(x-h)}{h^2} = W''(x) + \underbrace{O(h^2)}_{\substack{\text{includes derivatives} \\ W^{(k)}(x)\ k \geq 4}}}$$

For W a polynomial (of certain degree), exact!

P2.

$v \in C^1(K)$

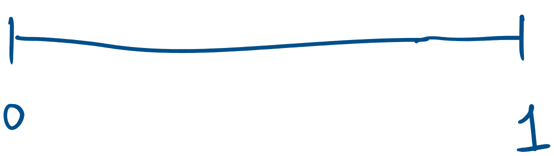


Square $\tilde{K}$

Maybe?: $\tilde{v} \overset{?}{\in} C^1(\hat{K})$ on E_i

idk? $\tilde{v}|_K = v$ $\quad \tilde{v}|_{E_i} = 0$

$$\max_x |v(x)| \leq C \|v\|_{H^1}$$



$$|v(0)| \leq C \|v\|_{H^1}$$

0 $\qquad\qquad\qquad\qquad$ 1

$$|v(0)| \le C\|v\|_{H^1}$$

$$|v(1)| \le C\|v\|_{H^1}$$

If $v(0)=0$ $\quad\rightsquigarrow\quad$ $\max_x |v(x)| \le C\|v'\|_{L^2}$

$$|v(1)| \le C\|v'\|_{L^2}$$

If $v(1)=0$

$$|v(0)| \le C\|v'\|_{L^2}$$

General v

$$v(x) = \underbrace{x\,v(x)}_{v_1(x)} + \underbrace{(1-x)\,v(x)}_{v_2(x)}$$

$$|v(1)| = |v_1(1)| \le C\|v_1'\|_{L^2}$$

$$v_1'(x) = v(x) + x\,v'(x)$$

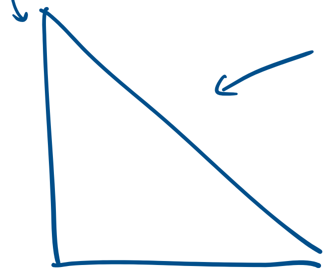
$$\rightsquigarrow \quad \|v_1'\|_{L^2} \le C\left(\|v\|_{L^2} + \|v'\|_{L^2}\right)$$

similar

$$\rightsquigarrow \quad |v(0)| = |v_2(0)| \le C\left(\|v\|_{L^2} + \|v'\|_{L^2}\right)$$

$\phi_1(x,y)$ $\quad x=0$ $\quad 1-x-y=0$

$\phi_1(x,y) = x$



$1 - x - y = 0$

$\phi_3(x,y) =$

$y = 0$

$\phi_2(x,y) = y$

$\phi_1 + \phi_2 + \phi_3 = 1$