610 Recitation 2024 03 20

Wednesday, March 20, 2024 7:33 AM

HW 4

1. Show $\exists\, C$ st when h small

$$\max_i |u_i^h| \le \underline{C}\, \underline{\|f\|_\infty}$$

In the hint, consider

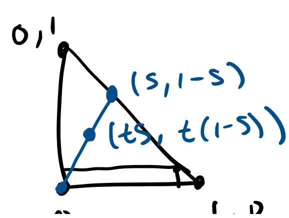
$$-\frac{v_{i+1}^h - 2v_i^h + v_{i-1}^h}{h^2} + b\,\frac{v_{i+1}^h - v_i^h}{h} = 1$$

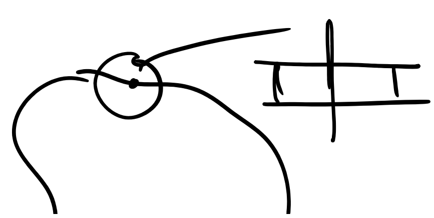
$$\rightsquigarrow \quad \max_i |u_i^h| \le \underbrace{\left(\max_i v_i^h\right)}_{\text{still depends on } h!} \|f\|_\infty$$

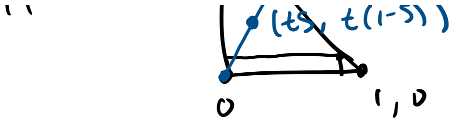
To finish, show $\exists\, C$ st when h is small

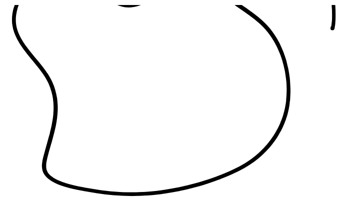
$$\max_i v_i^h \le C$$

Possible to do, but a little painful.

2. $K =$ 







$$\varphi(s,t) = (ts, t(1-s))$$

parameterizes K with $Q = [0,1]^2$

$$\int_K f(x,y)\,dx\,dy \underset{\substack{\uparrow \\ (x,y)=\varphi(s,t)}}{=} \int_0^1\int_0^1 f(\varphi(s,t))\,\underbrace{|\det D\varphi(s,t)|}_{t}\,ds\,dt$$

$$\det D\varphi(s,t) = \det\begin{pmatrix} t & s \\ -t & 1-s \end{pmatrix} = \det\begin{pmatrix} t & s \\ 0 & 1 \end{pmatrix}$$

$$= t$$

Idea: realize

$$\underbrace{\int_0^1 v(s,1-s)^2\,ds}_{\|v\|^2_{L^2(e)}} = \int_0^1\int_0^1 \frac{d}{dt}\left(t^2 v(\underbrace{ts,t(1-s)}_{\varphi(s,t)})^2\right) dt\,ds$$

$$= \int_0^1\int_0^1 2t\, v(ts,t(1-s))^2\,dt\,ds +$$

$$t^{\cancel{2}\,1}\, 2v(ts,t(1-s)) \cdot \{Dv(\varphi(s,t)) \cdot \underbrace{(ts,t(1-s))}_{\varphi(s,t)}\}\,dt\,ds$$

$$2\|v\|^2_{L^2(K)} + 2\underbrace{\int_K v(x,y)\,\big(\nabla v(x,y)\cdot(x,y)\big)\,dx\,dy}_{C-S\ \ldots}$$

HW5 P1

V, W V.S.

$$V\times W = \{(v,w) \mid v\in V,\ w\in W\}$$

$$\underline{\alpha(v_1,w_1)+(v_2,w_2)} = (\alpha v_1+v_2,\ \alpha w_1+w_2)$$

$$a : (\underbrace{V\times W}_{X}) \times (\underbrace{V\times W}_{X}) \longrightarrow \mathbb{R} \quad \underline{\text{bilinear}}$$

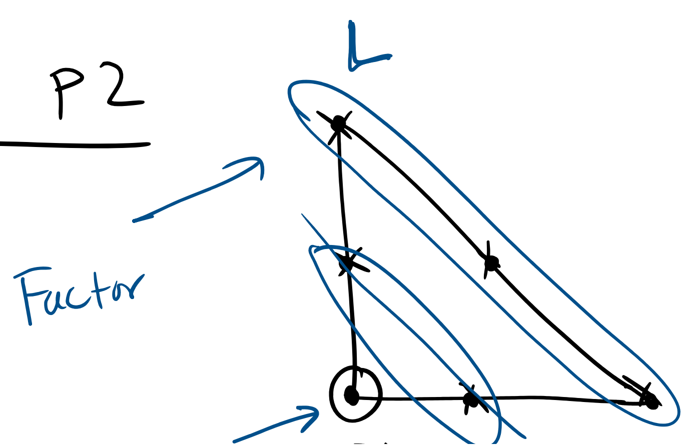
$X \ni x=(v,w)$

$$a\big((v_1,w_1),\ \alpha\underbrace{(v_2,w_2)}+(v_3,w_3)\big) = \alpha\, a\big((v_1,w_1),(v_2,w_2)\big) + a\big((v_1,w_1),(v_3,w_3)\big)$$

fix one $\longrightarrow$ linear in other

and vice-versa

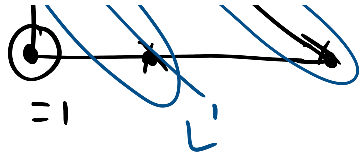
HW5 P2



$\varphi_i \in \mathbb{P}^2$ st

$\sigma_j(\varphi_i) = \delta_{ij}$

$\updownarrow$



$\varphi_i(\text{point}) = 1$

$\varphi_i(\text{other points}) = 0$

$$\varphi_i = \underbrace{L(x,y)}_{\neq 0 \text{ on } L'} \psi_i \longrightarrow = 0 \text{ at 2 pts on } L'$$

d $= 0$ on 2 points on L'

ψ_i: deg $\leq 1 \longrightarrow \psi_i = 0$ on L'

Factor!

$$\psi_i = L'(x,y)\, C$$

$$\varphi_i = L(x,y)\, L'(x,y)\, C$$